



# Partial difference equation based model reference control of a multiagent network of underactuated aquatic vehicles with strongly nonlinear dynamics<sup>☆</sup>

Jun Y. Kim<sup>a</sup>, Vivek Natarajan<sup>a</sup>, Scott D. Kelly<sup>b</sup>, Joseph Bentsman<sup>a,\*</sup>

<sup>a</sup> Department of Mech. Sci. and Eng., University of Illinois at Urbana-Champaign, 1206 W Green Street, Urbana, IL 61801, United States

<sup>b</sup> Department of Mech. Eng. and Eng. Sci., The University of North Carolina at Charlotte, 9201 University City Blvd, Charlotte, NC 28223-0001, United States

## ARTICLE INFO

### Article history:

Received 11 March 2009

Accepted 3 December 2009

### Keywords:

Model reference control

Partial differential equations

Partial difference equations

Multiagent systems

## ABSTRACT

In a recent work, the authors presented an extension of robust model reference adaptive control (MRAC) laws for spatially varying partial differential equations (PDEs) proposed by them earlier for the decentralized adaptive control of heterogeneous multiagent networks with agent parameter uncertainty using the partial difference equations (PdEs) on graphs framework. The examples provided demonstrated the capabilities of this approach under the assumption that individual vehicles executing coordinated maneuvers were fully actuated and characterized by linear dynamics. However, detailed models for autonomous vehicles – whether terrestrial, aerial, or aquatic – are often underactuated and strongly nonlinear. Using this approach, but assuming the plant parameters to be known, this work presents the model reference (MR) control laws without adaptation for the coordination of underactuated aquatic vehicles modeled individually in terms of strongly nonlinear dynamic equations arising from ideal planar hydrodynamics. The case of unknown plant parameters for this class of underactuated agents with complex dynamics is an open problem. The paper is based on an invited talk on adaptive control presented at the 2008 World Congress of Nonlinear Analysts.

© 2009 Elsevier Ltd. All rights reserved.

## 1. Introduction

The coordination of aquatic vehicles is a topic of growing interest in the context of applications ranging from military reconnaissance to environmental sampling [1]. However, an underwater vehicle that mimics locomotion of fish is an underactuated system, thereby posing additional challenges for designing control laws capable of enforcing its reasonable tracking capability. Tracking control of a single vehicle using backstepping is presented in [2] and references therein. However, designing a trajectory following control law for an underactuated aquatic vehicle typically requires lengthy algebra and the resulting control laws are rather involved. Control of a network of such vehicles presents additional difficulties.

The latter difficulty can be in part addressed by extending control laws designed for systems represented by partial differential equations to multiagent systems. This is accomplished using the framework of PdEs on graphs recently introduced in [3]. As indicated in [3], if parabolic or hyperbolic PDEs are made to exhibit a desirable dynamics (cf. [4] or [5]), PdEs on graphs defined through Laplacian operators essentially inherit this dynamics under similar conditions. The application of this setting to multiagent systems is carried out in [6] and [7]. Recently, using this framework the authors

<sup>☆</sup> This work was jointly supported by the National Science Foundation under grants CMS-0324630, ECS-0501407, and CMS-0449319.

\* Corresponding author. Tel.: +1 217 244 1076; fax: +1 217 244 6534.

E-mail address: [jbentsma@illinois.edu](mailto:jbentsma@illinois.edu) (J. Bentsman).

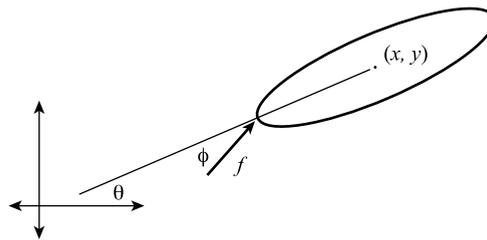


Fig. 1. Simplified model for an underwater vehicle.

presented in [8] an extension of robust MRAC laws for spatially varying PDEs under distributed sensing and actuation proposed in [9] and [10] to the decentralized adaptive control of heterogeneous multiagent networks with fully actuated agents under agent parameter uncertainty.

Using the approach of [8] and assuming no parameter uncertainty, this work presents the model reference (MR) control laws that address position tracking by means of a network of underactuated aquatic vehicles modeled in terms of ideal planar hydrodynamics. Instead of trajectory following, a reference model characterized by a simpler dynamics is specified, and the control objective is to steer the system to track the model. In [11] the reference model for systems with double-integrator dynamics was chosen to imitate the dynamics of an undamped wave equation, whereas in [12] the control law was designed so as to make the plant behave like a strongly damped wave equation. Here we choose a reference model that mimics the dynamics of a viscously damped wave equation.

The reference model generates a separate reference trajectory for each vehicle using a distinct reference input for each of them. It is reasonable to presume that only some of the vehicles, referred to as *leaders*, are equipped with sophisticated sensing instrumentation and hence are capable of determining their particular desired trajectories. The reference input for the *leaders* should hence be designed so that the reference trajectory generated by the reference model for the *leaders* corresponds to the desired trajectory. This design issue has been dealt with in detail in [8]. Hence in this work we assume directly that the reference inputs for the *leaders* are specified. The remaining vehicles are referred to as *followers*. The reference input for the followers is always set to zero. The reference trajectory for the followers is determined by those of the leaders through their coupling in the reference model. The coupling is chosen so that the reference trajectory of each follower is affected only by the reference trajectory of the neighboring vehicles. This ensures that the implementation of the control law involves communication between neighboring agents only, hence making the configuration decentralized.

## 2. Individual vehicle dynamics

### 2.1. Dynamics of a free elliptical body in an ideal fluid

In the present work each individual vehicle is modelled as an elliptical body moving in an otherwise quiescent infinite ideal fluid in the plane. The dynamics of such a body is described in many standard hydrodynamics texts, including [13], and is equivalent to the dynamics of a rigid planar body in space with an *effective mass* that depends on the body's direction of motion. An elliptical body, in particular, exhibits a larger effective mass for the lateral translation than for the longitudinal one. Control is introduced into the model in the form of an external force with magnitude  $f$  applied to the rearmost point on the body and a direction relative to the body's longitudinal axis specified by the angle  $\phi$ , as shown in Fig. 1. The input force is defined relative to a body-fixed frame to represent the influence of a thruster or a propulsive appendage affixed to the body.

The position of the vehicle is characterized by the  $x, y$ -coordinates of its center of mass and a  $\theta$ -coordinate specifying its orientation, as shown in Fig. 1. The equations of motion for the forced system may be obtained through straightforward application of the integral Lagrange–d'Alembert principle [14], defining the Lagrangian for the system as the total kinetic energy of the body and the surrounding fluid. If  $2a$ ,  $2b$ , and  $\rho_e$  are the length, the width, and the density of the body, respectively, and  $\rho_f$  is the density of the fluid, then the principal effective inertias of the body are given by

$$\mu = ab\pi\rho_e + b^2\pi\rho_f, \quad \nu = ab\pi\rho_e + a^2\pi\rho_f,$$

and

$$\iota = \frac{1}{4}ab(a^2 + b^2)\pi\rho_e + (a^2 - b^2)^2\rho_f.$$

Defining  $\kappa = \mu + \nu$  and  $\lambda = \mu - \nu$ , the equations governing the dynamics of a single vehicle take a relatively compact form:

$$\begin{bmatrix} \kappa\ddot{x} & \dot{y} - 2\dot{x}\dot{\theta} & 2\dot{y}\dot{\theta} + \ddot{x} \\ \kappa\dot{y} & 2\dot{y}\dot{\theta} + \ddot{x} & 2\dot{x}\dot{\theta} - \dot{y} \\ \iota\ddot{\theta} & \frac{1}{2}(\dot{x}^2 - \dot{y}^2) & -\dot{x}\dot{y} \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \sin 2\theta \\ \lambda \cos 2\theta \end{bmatrix} = \begin{bmatrix} 2 \cos(\phi + \theta) \\ 2 \sin(\phi + \theta) \\ -a \sin \phi \end{bmatrix} f. \quad (1)$$

Download English Version:

<https://daneshyari.com/en/article/1713662>

Download Persian Version:

<https://daneshyari.com/article/1713662>

[Daneshyari.com](https://daneshyari.com)