Nonlinear Analysis: Hybrid Systems 5 (2011) 394–406

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

About the stochastic and continuous Petri nets equivalence in the long run

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ARTICLE INFO

Article history:
Received 25 June 2010
Accepted 14 February 2011

Keywords:
Stochastic Petri nets
Timed continuous Petri nets
Piecewise-affine hybrid systems
Fluidification
Reliability analysis

ABSTRACT

Reliability analysis is often based on stochastic discrete event models like stochastic Petri nets (SPNs). For large dynamical systems with numerous components, the analytical expression of the SPNs steady state is full of complexities because of the combinatory explosion with discrete models. Moreover, the estimation of mean markings thanks to simulations is time consuming in case of rare events. For these reasons, Petri net fluidification may be an interesting alternative to provide a reasonable estimate of the asymptotic behavior of stochastic processes. Unfortunately, the steady states of SPNs and timed continuous Petri nets (contPNs) with the same structure, same initial marking and same firing rates are mainly often different. The region of SPN steady states (when firing rates are defined in a polyhedral area) contrasts with that of contPN ones. The purpose of this paper is to illuminate this issue in taking advantage of the piecewise-affine hybrid structure of contPNs. Regions and critical regions are defined in the marking space in order to characterize this structure. Based on this characterization, the main contribution is to propose a transformation of the considered SPN into a contPN with the same structure, modified firing rates and homothetic initial marking so that the corrected contPN converges partially to the same mean marking than the SPN. Consequently, a global understanding of an SPN steady state can be obtained according to the corrected contPN.

1. Introduction

Reliability analysis is a major challenge to improve the safety of industrial processes. For complex dynamical systems with numerous interdependent components, such studies are mainly based on stochastic discrete event models like stochastic Petri nets (SPNs) [1,2]. Such models lead to analytical results and also to numerical simulations. But in case of large systems, the combinatory explosion limits the use of analytical results and the simulation with SPNs may also be slow due to the possible occurrence of rare events. In this context, partial or complete fluidification of the discrete models can be discussed as a relaxation method. Several extensions of SPNs have been proposed for that purpose. Such models are hybrid systems. Timed continuous Petri nets under infinite server semantic (contPNs) are piecewise-affine hybrid systems that approximate the discrete nature of Timed Petri nets [3,4]. Markovian and hybrid Markovian Petri nets have also been introduced [5,6]. Such models, based on contPNs, include additive Gaussian variables to approximate SPNs. Fluid stochastic Petri nets are another extension of hybrid nets with discrete and continuous portions that may affect each other [7]. Finally, hybrid adaptive Petri nets have also been introduced with transitions that may behave in a continuous or discrete mode [8]. The previous hybrid fluid models are useful for performance evaluation and reliability analysis but provide only approximations of the stochastic processes under consideration.

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1751-570X/$ – see front matter
doi:10.1016/j.nahs.2011.02.001

This paper continues the investigation of stochastic systems with contPNs. It is devoted to the equivalence of SPNs and contPNs in the long run (i.e. same asymptotic mean markings and average throughputs). In our preceding works, we have pointed out the limits of the trivial fluidification of SPNs (i.e. contPNs with the same net structure, same initial marking and same transition firing rates). Sufficient conditions for equivalence in the long run have been discussed according to the partition in regions of the marking space and to the presence of critical regions. Characterizations of the regions have been proposed in full and reduced state spaces [9,10]. The particular case of mono T-semiflow PNs has been also investigated [11]. Then, contPNs with piecewise-constant firing rates have been proposed [12] for long run equivalence in non-critical regions. The main contribution of this paper is to define contPNs for long run equivalence also in critical regions. Homothetic transformations of the initial marking and modified firing rates are considered so that the corrected contPNs converge partially to the exact steady state of SPNs. In addition, in comparison with our preceding works [11,9,10,12] the firing rates are no longer defined as piecewise-constant functions but as constant parameters. The proposed results are helpful in the sense that they provide a better global understanding of SPNs steady state when firing rates vary in a polyhedral region. They can be used to work out continuous models that have distributions of steady states similar to the ones of SPNs. As a consequence, the resulting models could be used to evaluate the performances of stochastic or hybrid stochastic systems. Reliability issues are a concern at first. But they could also be used to design observers or controllers according to the proposed class of contPNs [13,14].

The paper is organized as follows. Section 2 presents stochastic and timed continuous Petri nets. Section 3 introduces reduced state variables and contains a characterization of contPNs regions in reduced and full state spaces. Section 4 is devoted to the fluidification of SPNs. The limitations of trivial fluidification are discussed. Then, sufficient conditions for local and global long run equivalences are established. Section 5 presents two examples and Section 6 provides some conclusions.

2. Petri nets

2.1. Notations

A Petri net (PN) is defined as $< \boldsymbol{P}, \boldsymbol{T}, W_{PR}, W_{PO} >$ where $\boldsymbol{P} = \{P_i\}$ is a set of n places and $\boldsymbol{T} = \{T_j\}$ is a set of q transitions, $W = W_{PO} - W_{PR} \in (\mathbf{Z})^{n\times q}$ is the incidence matrix, $M(t)$ is the PN marking vector with $t \geq 0$ the time variable and M_I the PN initial marking [3]. Each transition T_j fires according to the round towards zero (i.e. "floor" function) of its enabling degree $enab_j(M(t))$:

$$enab_j(M(t)) = \min\{m_k(t)/w_{kj}^{PR}, P_k \in {}^{\circ}\boldsymbol{T}_j\} \tag{1}$$

where ${}^{\circ}\boldsymbol{T_j}$ stands for the set of T_j upstream places. A firing sequence σ is defined as an ordered series of transitions that successively fire from initial marking M_I to marking M (i.e. $[M_I\sigma > M)$. Such a sequence may be represented by the PN firing count vector $\sigma(t) = (\sigma_j(t)),\ j = 1, \ldots, q$. $\sigma(t)$ is an application from $\mathbf{R}^+$ to $(\mathbf{Z}^+)^q$ such that, for any $t \geq 0, \sigma_j(t)$ is the number of T_j firings from initial time to t.

PNs may have P-semiflows and T semiflows. A P-semiflow $y \in (\mathbf{Z}^+)^n$ is a non-zero solution of equation $y^T.W = 0$ (i.e. y^T is a left annuller of W). A Petri net is conservative if a P-semiflow y exists that covers all places (i.e. $y > 0$). Let us define $Y \in (\mathbf{Z}^+)^{n\times hp} = (y_1|\ldots|y_{hp})$ as the matrix obtained from the hp minimal P-semiflows $y_i, i = 1, \ldots, hp$ of W (i.e. y_i is not a proper superset of the support of other P-semiflows). Y is of rank hp and satisfies Eq. (2):

$$Y^T.M(t) = C, \quad t \geq 0 \tag{2}$$

with $C = Y^T.M_I$. A T-semiflow $z \in (\mathbf{Z}^+)^q$ is a non-zero solution of equation $W.z = 0$ (i.e. z is a right annuller of W). A Petri net is consistent if a T-semiflow z exists that affects all transitions (i.e. $z > 0$). Let us define $Z \in (\mathbf{Z}^+)^{n\times ht} = (z_1|\ldots|z_{ht})$ as the matrix of rank ht obtained from the ht minimal T-semiflows $z_j, j = 1, \ldots, ht$ of W (i.e. z_j is not a proper superset of the support of other T-semiflows). Join free (JF) nets are PNs such that $|{}^{\circ}\boldsymbol{T}_j| = 1, j = 1, \ldots, q$, and choice free (CF) nets are PNs such that $|\boldsymbol{P}_i{}^{\circ}| = 1, i = 1, \ldots, n$. Conservative JF nets are mono P-semiflow (MPS-PN) and consistent CF nets are mono T-semiflow (MTS-PN) [15].

PN1 and PN2 [16], described in Fig. 1 are two examples of consistent PNs. PN1 has 2 joins on T_1 and T_2 and a single choice according to P_4. The incidence matrices are given by W_{PR1} and W_{PO1}:

$$W_{PR1} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}, \qquad W_{PO1} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

PN1 has 2 P-semiflows: $y_{11} = (1\ 1\ 1\ 0)^T, y_{12} = (1\ 0\ 4\ 1)^T$, a single T-semiflow $z_1 = (1\ 1\ 1)^T$ and one can define $Y(PN1) = (y_{11}y_{12}) \in (\mathbf{Z}^+)^{5\times 2}, Z(PN1) = (z_1) \in (\mathbf{Z}^+)^{4\times 1}$ and $C(PN1) = (17\ 19)^T$. The P-semiflow y_{11} means that the global marking $m_1 + m_2 + m_3$ in the group of places P_1, P_2 and P_3 is constant. Similarly the T-semiflow z_1 means that the successive firings of transitions T_1 to T_4 let the marking unchanged.

PN2 has also 2 joins on T_1 and T_2 and a single choice according to P_1. The incidence matrices are given by W_{PR2} and W_{PO2}. PN2 has 2 P-semiflows: $y_{21} = (0\ 0\ 0\ 1\ 1)^T, y_{22} = (1\ 1\ 2\ 1\ 0)^T$ and a single T-semiflow $z_2 = (1\ 1\ 1\ 1)^T$. Let us define $Y(PN2) = (y_{21}y_{22}) \in (\mathbf{Z}^+)^{5\times 2}, Z(PN2) = (z_2) \in (\mathbf{Z}^+)^{4\times 1}$ and $C(PN2) = (4\ 5)^T$.