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Reliable H_∞ control for a class of switched nonlinear systems with actuator failures[☆]

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Abstract

This paper focuses on the problem of reliable H_∞ control for a class of switched nonlinear systems with actuator failures among a prespecified subset of actuators. We consider the case in which the never failed actuators cannot stabilize the system. The multiple-Lyapunov-function method is exploited to derive a sufficient condition for the switched nonlinear system to be asymptotically stable with H_∞-norm bound. This condition is given in the form of a set of partial differential inequalities. As a special application, a hybrid state feedback strategy is proposed to solve the standard H_∞ control problem for non-switched nonlinear systems when no single continuous controller is effective.

Keywords: Switched systems; Reliable H_∞ control; Multiple Lyapunov functions; Actuator failures; LaSalle's invariance principle

1. Introduction

Recent years have witnessed rapidly growing interest in switched systems which are an important class of hybrid systems [1–7]. The motivation for studying switched systems is from the fact that many practical systems are inherently multi-models in the sense that several dynamical subsystems are required to describe their behavior depending on various changing environmental factors. The widespread applications of switched systems are also motivated by increasing performance requirements in control. In the study of stability analysis for switched systems, the multiple-Lyapunov-function approach has been shown to be an effective tool [8–12]. For example, in [11], a hybrid nonlinear control methodology is presented by using multiple Lyapunov functions for a class of switched nonlinear systems with input constraints. On the basis of multiple Lyapunov functions, the problem of H_∞ control for switched nonlinear systems is addressed in [12].

On the other hand, owing to the growing demands of system reliability in aerospace and industrial process, the study of reliable control has recently attracted considerable attention. Classical H_∞ control methods may not provide satisfactory performance because failures of control components often occur in the real world. Therefore, to overcome

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this problem, reliable H_∞ control has made great progress recently [13–17]. In particular, in [13], a methodology for the design of a reliable H_∞ controller for the case of actuator failures is presented. In [15], the reliable H_∞ control problem for affine nonlinear systems is solved by using the Hamilton–Jacobi inequality approach for the case of actuator and sensor failures. However, these reliable H_∞ design methods are all based on a basic assumption that the never failed actuators can stabilize the given system. For the case where actuators suffer "serious failure" — the never failed actuators cannot stabilize the given system, these design methods of existing reliable H_∞ control do not work.

In this paper, we introduce a switching strategy to solve the reliable H_∞ control problem for switched nonlinear systems with "serious failed" actuators. On the basis of the multiple-Lyapunov-function technique, a sufficient condition for the switched nonlinear systems to be asymptotically stable with H_∞-norm bound is derived for all admissible actuator failures. Furthermore, as a direct application, a hybrid state feedback strategy is proposed to solve the standard H_∞ control problem for nonlinear systems when no single continuous controller is effective. Due to the complexity of switched systems, there are few results on the reliable control for switched systems [18,19]. Compared with the existing results on the reliable control for switched systems, the results of this paper have three features. Firstly, we consider the worst case where the never failed actuators cannot stabilize the system, while the existing works only aimed to address the case where never failed actuators are adequate to stabilize the system. Second, the reliable control problem is solved by using the multiple-Lyapunov-function technique, while the existing literature used the common Lyapunov function method. Third, the problem of H_∞ control is solved in this paper while the existing works usually considered the problem of stability.

2. Problem formulation

Consider the switched nonlinear systems described by a state-space model of the form

$$\begin{aligned} \dot{x} &= f_\sigma(x) + g_\sigma(x)u_\sigma + p_\sigma(x)w_\sigma, \\ z &= \begin{pmatrix} h_\sigma \\ u_\sigma \end{pmatrix}, \end{aligned} \tag{1}$$

where $\sigma : R_+ \to M = \{1, 2, \ldots, m\}$ is the switching signal to be designed, $x \in R^n$ is the state, $u_i = (u_{i1}, \ldots u_{im_i})^T \in R^{m_i}$ and $w_i = (w_{i1}, \ldots w_{iq_i})^T \in R^{q_i}$ denote the control input and disturbance input of the i-th subsystem respectively, z is the output to be regulated. Further, let $f_i(x) \in R^n$, $g_i(x) = (g_{i1}(x), \ldots g_{im_i}(x)) \in R^{n\times m_i}$, $p_i(x) = (p_{i1}(x), \ldots p_{iq_i}(x)) \in R^{n\times q_i}$, $h_i(x) = (h_{i1}(x), \ldots, h_{ip_i}(x))^T \in R^{p_i}$, $f_i(0) = 0, h_i(0) = 0, i = 1, 2, \ldots, m$.

We adopt the following notation from [8]. A switching sequence is expressed by

$$\Sigma = \{x_0; (i_0, t_0), (i_1, t_1), \ldots, (i_j, t_j), \ldots, | i_j \in M, j \in N\}$$

in which t_0 is the initial time, x_0 is the initial state, (i_k, j_k) means that the i_k-th subsystem is activated for $t \in [t_k, t_{k+1})$. Therefore, when $t \in [t_k, t_{k+1})$, the trajectory of the switched system (1) is produced by the i_k-th subsystem. For any $j \in M$,

$$\Sigma_t(j) = \{[t_{j_1}, t_{j_1+1}), [t_{j_2}, t_{j_2+1}), \ldots [t_{j_n}, t_{j_n+1}) \ldots, \sigma(t) = j, t_{j_k} \le t < t_{j_k+1}, k \in N\} \tag{2}$$

denotes the sequence of switching times of the j-th subsystem, in which the j-th subsystem is switched on at t_{j_k} and switched off at t_{j_k+1}.

We classify actuators of the i-th subsystem into two groups. One is a set of actuators susceptible to failures, denoted by $\Theta_i \subseteq \{1, 2, \ldots, m_i\}, i \in M$. The other is a set of actuators robust to failures, denoted by $\bar{\Theta}_i \subseteq \{1, 2, \ldots, m_i\} - \Theta_i, i \in M$. For $\omega_i \subseteq \Theta_i$, introduce the decomposition

$$g_i(x) = g_{\omega_i}(x) + g_{\bar{\omega}_i}(x),$$

where

$$g_{\omega_i}(x) = (\delta_{\omega_i}(1)g_{i1}(x), \delta_{\omega_i}(2)g_{i2}(x), \ldots, \delta_{\omega_i}(m_i)g_{im_i}(x))$$