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Stabilization of string system with linear boundary feedback☆

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Abstract

In this paper we consider the uniform stabilization of a vibrating string with Neumann-type boundary conditions. Herein we do not consider a controller stabilizing the system, but emphasize the simplicity and effectiveness of the controller. We adopt the linear feedback control law, which comprises both boundary velocity and position, and prove that the closed loop system is dissipative and asymptotically stable. By asymptotic analysis of frequency of the closed loop system, we give asymptotic expression of the frequencies and the Riesz basis property of eigenvectors and generalized eigenvectors of the system operator under some conditions, and hence obtain the exponential stability of the closed loop system. We show that, for a particular case, the system may be super-stable in subspace of a codimensional one. From the above result, we conclude that one can design a much simpler linear controller by choice of parameters such that the closed loop system is of Riesz basic properties and exponentially stable.

Keywords: Vibrating string equation; Stabilization; Spectrum; Riesz basis

1. Introduction

In the present paper, we consider the control problem of a vibrating string:

$$\begin{cases} w_{tt}(x,t) = w_{xx}(x,t) - \nu(x)w(x,t), & 0 < x < 1,\ t > 0, \\ w_x(0,t) = u_1(t), \qquad w_x(1,t) = u_2(t), & t > 0, \\ w(x,0) = w_0(x), \qquad w_t(x,0) = w_1(x) \end{cases} \tag{1.1}$$

where $w(x,t)$ denotes the transversal displacement of the string depart from its equilibrium position at x and time t, $\nu(x) \geq 0$ is a potential function, and $u_j(t)$, $j = 1, 2$, are the control input. This model is a particular case of the modelling of a flexible torque arm in [4]

$$\begin{cases} w_{tt}(x,t) - (aw_x)_x(x,t) + \alpha w_t(x,t) + \beta w(x,t) = 0, & x \in (0,1),\ t > 0, \\ (aw)_x(0,t) = u_1(t), & t > 0, \\ (aw)_x(1,t) = u_2(t), & t > 0 \end{cases} \tag{1.2}$$

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where

$$\begin{cases} \alpha \geq 0, & \beta \geq 0, \\ a(x) \in W^{1,\infty}(0,1), & a(x) \geq a_0 > 0, \ \forall x \in [0,1]. \end{cases} \tag{1.3}$$

Note that if the system has no viscous damping, i.e. $\alpha = 0$, then (1.2) becomes (1.1) under a change of variable. So (1.1) is essentially equivalent to the variable coefficient case. For such a system, an important task is to determine feedback control inputs $u_1(t)$ and $u_2(t)$ such that the closed loop system is asymptotically or uniformly stable, i.e. $(w, w_t) \to 0$, as $t \to \infty$, in some function space.

For the system (1.2), the boundary stabilization problem has been studied by many authors. For example, in the case that $\beta = 0$, Conrad and Rao in [3] has proved that the feedback control law:

$$\begin{cases} u_1(t) = \alpha w(0,t) + F(w_t(0,t)), \\ u_2(t) = -\alpha w(1,t) - F(w_t(1,t)), \quad \alpha > 0 \end{cases}$$

stabilizes asymptotically the system under a suitable choice of the function F; Rao in [9] obtained the stabilization result with the control law:

$$\begin{cases} u_1(t) = \gamma w(0,t) + F(w_t(0,t)), \\ u_2(t) = -M w_{tt}, \quad M, \gamma > 0. \end{cases}$$

In [4], a stabilization result was obtained by the feedback:

$$\begin{cases} u_1(t) = k_p w(0,t) + k_v w_t(0,t) + \displaystyle\int_0^1 G(x) w(x,t)\mathrm{d}x + \frac{k_v}{k_p}\int_0^1 G(x) w_t(x,t)\mathrm{d}x, \\ u_2(t) = 0, \end{cases}$$

with $k_p, k_v > 0$ and $G(x)$ is a function in $L^2[0,1]$. Moreover, Mifdal in [7] proved that the control law:

$$\begin{cases} u_1(t) = m w_{tt}(0,t) + \alpha\beta w_t(0,t) - \gamma w_{xt}(0,t) \\ u_2(t) = -M w_{tt}(1,t), \end{cases}$$

or

$$\begin{cases} u_1(t) = m w_{tt}(0,t) + (\beta + \alpha\gamma) w_t(0,t) - \gamma w_{xt}(0,t) \\ u_2(t) = -M w_{tt}(1,t), \end{cases}$$

may stabilize uniformly the system.

For $\beta \neq 0$, there are many authors studying the stabilization problem of (1.2). Here we refer to recent work [2] and the references therein. In [2], the authors proved that if $\beta > 0$ and $\alpha \geq 0$ in (1.2), the system is asymptotically stabilized by the nonlinear feedback law:

$$\begin{cases} u_1(t) = f(w_t(0,t)), \\ u_2(t) = g(w_t(1,t)) \end{cases}$$

where f and g are suitable functions.

All references mentioned above show that the feedback control law should contain the boundary velocity. However, it seems to be not sufficient in some cases that the control law contains merely the boundary velocity feedback. For instance, we take $a(x) \equiv 1$ and $\alpha = \beta = 0$ in the system (1.2), which is just the case $\nu(x) \equiv 0$ in (1.1), the velocity feedback law:

$$u_1(t) = k_1 w_t(0,t), \qquad u_2(t) = -k_2 w_t(1,t) \tag{1.4}$$

cannot compel the system back to its equilibrium position because 0 is an eigenvalue of the system.

In control theory it is important to design simple and effective controllers. The design of the nonlinear controllers seems too complicated. Our idea is to find a linear feedback controller that can stabilize uniformly the system (1.1). Recently, Kobayashi in [5] obtained the asymptotic stability of the system (1.1) with $\nu(x) \equiv 0$ by using controller:

$$u_1(t) = 0, \qquad u_2(t) = -k_2[y(t) + \xi(t)]$$