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Limit cycles analysis of reset control systems with reset band

Alfonso Baños [a,*], Sebastián Dormido [b], Antonio Barreiro [c]

[a] Dpt. Informática y Sistemas, Univ. Murcia, 30071 Murcia, Spain

[b] Dpt. Informática y Automática, U.N.E.D., 28040 Madrid, Spain

[c] Dpt. Ingeniería de Sistemas y Automática, Univ. Vigo, 36200 Vigo, Spain

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ABSTRACT

The reset band is a simple idea, and a must in practice, to improve reset compensation by adding extra phase lead in a feedback loop. However, a formal treatment of how the reset band can affect stability and performance of a reset control system is still an open issue. This work approaches the problem of the existence and stability of limit cycles of reset control systems with reset band. A frequency domain approach is given by using standard methods based on the describing function. In addition, closed-form expressions have been obtained for the describing function of arbitrary order full reset compensators with and without reset band.

1. Introduction

Reset control is a kind of impulsive/hybrid control in which some of the compensator states are set to zero at those instants in which its input is zero. Using [1] or the most recent work [2], reset systems (without external inputs) can be classified as autonomous systems with impulse effects, an area of systems theory that has been started to be developed in the last few years. However, in the control literature the development of reset control systems dates back to the work of [3], and was followed by the works [4,5], where very simple reset compensators, CI (Clegg integrator) and FORE (first order reset element) respectively, were analyzed and design procedures were developed. More general reset compensators, including partial and full reset compensation, have been studied for example in [6].

One of the main difficulties of reset compensation is that closed-loop stability may be not guaranteed if reset actions are not properly performed, and in fact it is well known that reset can unstabilize a base stable control system. Thus, stability of reset control systems is a main concern from a theoretical and practical point of view, and several recent works have approached the stability problem. See for example [6–8], and the works [9–11].

An important practical issue of reset control is that compensator implementation is usually done by using reset band. In addition, it has been noted that use of reset band may improve stability and performance in systems with time-delays [12], due to the phase lead characteristic that is common to reset compensators. Phase lead can be even improved by using reset band. However, a formal analysis of how the reset band can affect stability and performance of a reset control system is still an open issue. In this work, a frequency domain approach by means of the describing function will be used to approach an important question such as the existence and stability of limit cycles in reset control systems with reset band. The analysis with the describing function will follow standard methods (see for example [13]). Although it is well-known that the describing function may fail in some cases, it has been proved to be an efficient method and has been formally justified in many practical cases [14,15], including some types of switched systems [16].

* Corresponding author.
E-mail addresses: abanos@um.es (A. Baños), sdormido@dia.uned.es (S. Dormido), abarreiro@uvigo.es (A. Barreiro).

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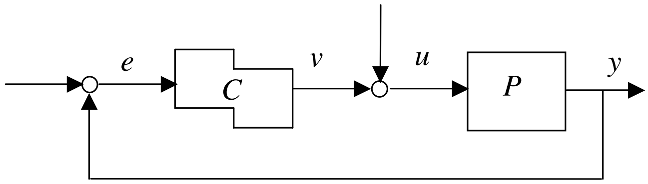

Fig. 1. Reset compensator C and LTI plant P.

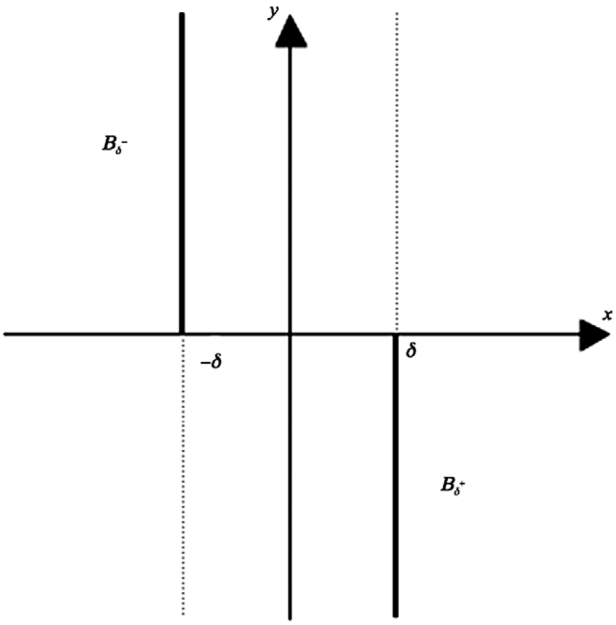

Fig. 2. Reset band.

The work is organized as follows. Section 2 introduces the problem setup, with a precise definition of a reset compensator with reset band. Note that single-input single-output systems are considered in this work. In Section 3, the describing function of reset control systems with reset band is calculated, in particular the describing function of CI and FORE with reset band are obtained. Section 4 approaches the existence and stability of limit cycles by using a describing function-based analysis, and finally in Section 5 the describing function method is justified by exploring the harmonics structure of the sinuosidal response and the low pass condition.

2. Reset compensators with reset band

The main motivation for the use of reset compensation is to improve the performance of a previously designed LTI control system, being the goal to reset some states of a base LTI compensator to improve control system performance, both in terms of velocity of response and relative stability. In general, these specifications will be impossible to achieve by means of LTI compensation.

Reset control systems with reset band are feedback control systems (Fig. 1), where for the purposes of this work the plant P is described in general by a transfer function $P(s)$, and the *reset* compensator C is given by the impulsive differential equation

$$C\colon \begin{cases} \dot{x}_r(t) &= A_r x_r(t) + B_r e(t), \quad (e(t), \dot{e}(t)) \notin B_\delta \\ x_r(t^+) &= A_\rho x_r(t), \qquad\qquad (e(t), \dot{e}(t)) \in B_\delta \\ v(t) &= C_r x_r(t) + D_r e(t) \end{cases} \tag{1}$$

where the reset band surface B_δ is given by $B_\delta = \{(x, y) \in \mathbf{R}^2 \mid (x = -\delta \wedge y > 0) \vee (x = \delta \wedge y < 0)\}$, being δ some non-negative real number. In this way, the compensator states are reset at the instants in which its input is entering into the reset band. In general, the reset band surface B_δ will consist of two reset lines B_δ^+ and B_δ^- in the plane, as shown in Fig. 2.

In the particular case $\delta = 0$, the *standard* reset compensator is obtained. On the other hand, if δ is big enough in relation to the error amplitude, then no reset action is produced and the reset compensator reduce to its base compensator, given by the transfer function $C_{base}(s) = C_r(sI - A_r)^{-1}B_r + D_r$.