Nonlinear Analysis: Hybrid Systems 6 (2012) 754–773

Contents lists available at SciVerse ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Robust observer design for nonlinear uncertain switched systems under asynchronous switching

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ARTICLE INFO

Article history:
Received 16 February 2011
Accepted 3 August 2011

Keywords:
Switched system
Observer
Average dwell time
Asynchronous switching
Eventual boundness

ABSTRACT

Switching between the system and the associated observer or controller is in fact asynchronous in switched control systems. However, many times we assume it synchronous, for simplicity. In this paper, the robust observer design problems for a class of nonlinear uncertain switched systems for synchronous and asynchronous switching are addressed. At first, a robust observer under synchronous switching is proposed based on average dwell time approach. After that, the results are extended to robust observer design in the asynchronous case. In this case, two working modes are adopted to facilitate the studies on the issue. Finally, an extension case covering more practical applications is investigated under asynchronous switching. The designed observer cannot maintain the asymptotical stability of error state, but the eventual boundness is guaranteed. At the end, a numerical design example is given to illustrate our results.

1. Introduction

Switched systems are an important class of hybrid systems found in the field of control theory and applications. They have attracted much attention of researchers and engineers in recent years. A switched system is composed of a family of continuous or discrete-time subsystems, described by differential or difference equations, along with a switching rule governing the switching between the subsystems. A switched system can be efficiently used to model many real-world systems which are inherently multi-model in the sense that several dynamical systems are required to describe their behavior. Many physical processes exhibit switched and hybrid behavior [1–3], and switching frequently occurs in many engineering applications, for example, in motor engine control [4], constrained robotics [5], networked control systems [6], etc. Furthermore, more and more industrial applications are considering switching strategy as an alternative, to improve control performance [7–10]. Generally, the stability and stabilization problems are the main concerns in the field of switched systems. Lyapunov stability and stabilization criteria have been emerged as the basic and effective governing rule in the field of switched systems [11–14]. Dwell time and average dwell time approaches have been employed to study the stability and stabilization of time-dependent switched systems [15–17]. For more details of the recent results on the basic problems in stability and stabilization for switched systems, the reader is referred to [18], and the references cited therein.

The state estimation problem has been investigated intensively by researchers, for many decades, in both continuous and discrete time domain, for control systems. After Luenberger proposed a design technique for an observer intended for linear time-invariant (LTI) systems in 1960s [19], numerous results on the Luenberger-like observer design for control systems were developed. Some results about the observer design for switched systems can also be found in the literature. Yet, most

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doi:10.1016/j.nahs.2011.08.001

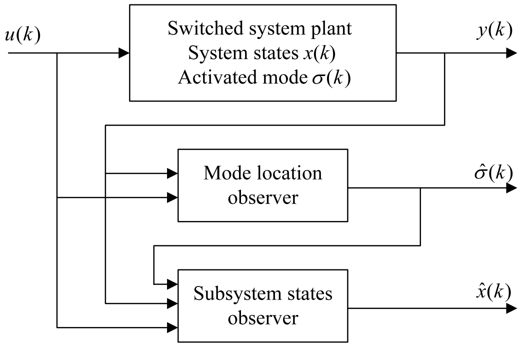

Fig. 1. A general scheme of observer for a switched system.

of them deal with switched linear systems. In [20,21], a common quadratic Lyapunov function (CQLF) which formulates the error state stability with any arbitrary switching signals is proposed. CQLF has been used to design an observer for switched continuous-time and discrete-time linear systems. An observer design method for a class of switched nonlinear systems is investigated in [22]. Pettersson presented an approach using multiple Lyapunov functions to design an observer for switched linear systems [23]. According to the proposed theory, the estimated state jumps on the switching surface to guarantee the convergence of error state estimation. However, the proposed approach in [23] can only be applied to the switched systems with switching sets described by linear hyper planes. Work presented in [24] deals with the nonlinear observer design for autonomous switching systems with jumps. Full and reduced order observers for a class of linear switched control systems are proposed in [25]. Here we present a general illustration of the observer for a switched system in Fig. 1.

In Fig. 1, we can see that the observer for a switched system is divided into two parts: the *Mode location observer* locating which subsystem is active, and, the *Subsystem state observer* estimating the state of the active subsystem. In our paper, we focus on the subsystem state observer design problem. Now, there exists a possibility that a wrong location mode is estimated, i.e. $\hat{\sigma}(k) \neq \sigma(k)$, during a certain period. The reason is that the mode location observer may need some time to identify the correct mode of plant. An obvious consequence is that the correct subsystem state observer cannot be activated in time such that the observer switching signal cannot match the practical system plant switching signal precisely due to the existence of the wrong mode location time interval. Because of this reason, the theory of asynchronous switching emerges.

As the reason given above, the controller or observer switching signal cannot match the system switching signal precisely in practice and the asynchronous switching inevitably exists. In fact, the necessity of taking into consideration the asynchronous switching is shown in efficient controller design in many mechanical and chemical systems [26,27] by determining the admissible delay of asynchronous switching. There are some results presented on control synthesis under asynchronous switching, such as the input-to-state stabilization [28], robust control for an uncertain switched system with time delay [29], stabilization for switched systems with perturbations [30], stability and $\mathcal{L}_2$ gain of switched linear systems [31,32], analysis and synthesis of Markov jump linear systems [33]. To the authors' knowledge, the robust observer design for a switched system under asynchronous switching have not been investigated fully. This is the main motivation of our study.

Synchronous switching between system and observer, where it is assumed that the mode location observer can detect the correct mode instantly, is difficult to be realized in practice. So the more realistic approach is asynchronous switching. The idea is to design a mode location observer identifying the right mode during a certain suitable interval. Furthermore, we will demonstrate that the case of synchronous switching is only a sub-class of more general case of asynchronous switching. In this paper, we first propose a robust observer under synchronous switching based on average dwell time approach. Then taking asynchronous switching into consideration, a robust observer is designed and it is found that the performance of the observer is insensitive to the length of asynchronous period. Finally, an extension case is given to accompany with more practical applications. The designed observer cannot maintain the asymptotic stability in the extension case, but the boundness of error state can be ensured and it is proved that the value of the boundary is related to the length of asynchronous period.

The remainder of this paper is organized as follows. In Section 2, some preliminary statements are presented, the robust observer under synchronous switching is proposed in Section 3. Then, the robust observer design in asynchronous case is investigated in Section 4. The extension case is considered in Section 5. A numerical example is given in Section 6. The results are concluded at the end in Section 7, with some guidelines for future work.

Notations: The notations used in this paper are fairly standard. The superscript "T" stands for matrix transposition. $\mathbb{R}^n$ denotes the n dimensional Euclidean space. $\mathbb{Z}^+$ represents the set of nonnegative integers. The notation $\|\cdot\|$ refers to the Euclidean norm. In symmetric block matrices, we use * as an ellipsis for the terms that are introduced by symmetry and diag$\{\cdots\}$ stands for a block-diagonal matrix. The notation $P > 0$ ($P \geq 0$) means P is real symmetric and positive definite (semi-positive definite). $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ stand for the smallest and the largest eigenvalue of matrix P. $\mathbf{I}$ stands for identity matrix. Notations "sup" and "inf" denote the supremum and infimum respectively.