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Bifurcation analysis and control of a class of hybrid biological economic models[☆]

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ABSTRACT

This paper systematically studies a hybrid predator–prey economic model, which is formulated by differential-difference-algebraic equations. It shows that this model exhibits two bifurcation phenomena at the intersampling instants. One is saddle–node bifurcation, and the other is singular induced bifurcation which indicates that economic profit may bring impulse at some critical value, i.e., rapid expansion of biological population in terms of ecological implications. On the other hand, for the sampling instants, the system undergoes Neimark–Sacker bifurcation at a critical value of economic profit, i.e., the increase of economic profit destabilizes the system and generates a unique closed invariant curve. Moreover, the state feedback controller is designed so that singular induced bifurcation and Neimark–Sacker bifurcation can be eliminated and the population can be driven to steady states by adjusting harvesting costs and the economic profit. At the same time, by using Matlab software, numerical simulations illustrate the effectiveness of the results obtained here.

1. Introduction

At present, mankind is facing the problems about shortage of resource and worsening environment. So there has been rapidly growing interest in the analysis and modelling of biological systems. From the view of human need, the exploitation of biological resources and harvest of population are commonly practiced in the fields of fishery, wildlife and forestry management. The predator–prey system plays an important and fundamental role among the relationships between the biological population. Many authors [1–5] have studied the dynamics of predator–prey models with harvesting, and obtained complex dynamic behavior, such as stability of equilibrium, Bogdanov–Takens bifurcation, Hopf bifurcation, limit cycle, heteroclinic bifurcation and so on. Quite a number of references [6–11] have discussed permanence, extinction and periodic solution of predator–prey models. In addition, there is also a considerable amount of literature on discrete dynamical systems, e.g., see [11–14] and references therein, modelling some species whose generations are non-overlapping by applying Euler scheme to differential equations. Most of these discussions on biological models are based on normal systems governed by differential equations or difference equations.

In daily life, economic profit is a very important factor for governments, merchants and even every citizen, so it is necessary to research biological economic systems, which can be described by differential–algebraic equations or differential-difference-algebraic equations. At present, most of differential algebraic equations can be found in the general power systems [15,16], economic administration [17], robot system [18], mechanical engineering [19] and so on. There

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are also several biological reports on differential algebraic equations [20–22]. Digital control systems of power grids [23] and biological systems, such as neural networks [24,25] and genetic networks [26], are typical models mixed with both continuous and discrete time sequences which can mathematically be formulated as differential-difference-algebraic equations. In addition, a lot of fundamental analyzing methods for differential algebraic equations and differential-difference-algebraic equations have been presented, such as local stability [27,28], optimal control [29], singular induced bifurcation [30] and so on. However, to the best of our knowledge, there are few reports on differential-difference-algebraic equations in biological fields. This paper mainly studies the stability and bifurcations of a new biological economic system formulated by differential-difference-algebraic equations. In what follows, we introduce the new biological economic system.

The basic model we consider is based on the following Lotka–Volterra predator–prey system with harvest

$$\begin{cases} \dfrac{d\tilde{x}}{d\tilde{t}} = r\tilde{x}\left(1 - \dfrac{\tilde{x}}{K}\right) - a\tilde{x}\tilde{y} - \tilde{E}\tilde{x}, \\ \dfrac{d\tilde{y}}{d\tilde{t}} = -\tilde{d}\tilde{y} + \tilde{b}\tilde{x}\tilde{y} - \tilde{E}\tilde{y}, \end{cases}$$

where $\tilde{x}$ and $\tilde{y}$ represent the prey density and predator density at time $\tilde{t}$, respectively. $r > 0$, $\tilde{d} > 0$ are the intrinsic growth rate of prey and the death rate of predator in the absence of food, respectively. $K > 0$ is the carrying capacity of prey. $a > 0$ and $\tilde{b} > 0$ measure the effect of the interaction of the two population. $\tilde{E}$ represents harvesting effort. $\tilde{E}\tilde{x}$ and $\tilde{E}\tilde{y}$ indicates that the harvests for prey and predator population are proportional to their densities at time $\tilde{t}$.

Combining the economic theory of fishery resource [31] proposed by Gordon in 1954, we can obtain a biological economic system expressed by differential algebraic equation

$$\begin{cases} \dfrac{d\tilde{x}}{d\tilde{t}} = r\tilde{x}\left(1 - \dfrac{\tilde{x}}{K}\right) - a\tilde{x}\tilde{y} - \tilde{E}\tilde{x}, \\ \dfrac{d\tilde{y}}{d\tilde{t}} = -\tilde{d}\tilde{y} + \tilde{b}\tilde{x}\tilde{y} - \tilde{E}\tilde{y}, \\ 0 = \tilde{E}(\tilde{p}_x\tilde{x} - \tilde{c}_x) + \tilde{E}(\tilde{p}_y\tilde{y} - \tilde{c}_y) - m, \end{cases} \tag{1}$$

where $\tilde{p}_x > 0$ and $\tilde{p}_y > 0$ are harvesting reward per unit harvesting effort for unit weight of prey and predator, respectively. $\tilde{c}_x > 0$ and $\tilde{c}_y > 0$ are harvesting cost per unit harvesting effort for prey and predator, respectively. $m \geq 0$ is the economic profit per unit harvesting effort. From a modelling view, more details of the algebraic equation can be found in the appendix.

When the populations have non-overlapping generations, many authors [32,33] have argued that the discrete time models governed by difference equations are more appropriate than the continuous ones. Suppose the predator population has no overlap between successive generations. Applying the Euler method to the predator equation of the system (1) with integral step size $\tilde{\tau}$, we have the following biological economic system expressed by differential-difference-algebraic equation

$$\begin{cases} \dfrac{d\tilde{x}}{d\tilde{t}} = r\tilde{x}\left(1 - \dfrac{\tilde{x}}{K}\right) - a\tilde{x}\tilde{y} - \tilde{E}\tilde{x}, \\ \tilde{y}_{n+1} = \tilde{y}_n + \tilde{\tau}(-\tilde{d}\tilde{y}_n + \tilde{b}\tilde{x}_n\tilde{y}_n - \tilde{E}_n\tilde{y}_n), \\ 0 = \tilde{E}(\tilde{p}_x\tilde{x} - \tilde{c}_x) + \tilde{E}_n(\tilde{p}_y\tilde{y}_n - \tilde{c}_y) - m. \end{cases} \tag{2}$$

We nondimensionalize the system (2) with the following scaling

$$x = \frac{\tilde{x}}{K}, \qquad y = \frac{a\tilde{y}}{r}, \qquad E = \frac{\tilde{E}}{r}, \qquad t = r\tilde{t}, \qquad \tau = r\tilde{\tau},$$

and then obtain the following biological economic system expressed by differential-difference-algebraic equation

$$\begin{cases} \dfrac{dx}{dt} = x(1 - x) - xy_n - Ex, \\ y_{n+1} = y_n + \tau(-dy_n + bx_ny_n - E_ny_n), \\ 0 = E(p_xx - c_x) + E_n(p_yy_n - c_y) - m, \end{cases} \tag{3}$$

where $x_n = x(n\tau), E_n = E(n\tau), y_n$ are the value at the instant $n\tau$ and x_n, y_n, E_n are constantly held during $n\tau \leq t < (n+1)\tau$ for the system (3). The non-dimensional parameters are defined as

$$d = \frac{\tilde{d}}{r}, \qquad b = \frac{\tilde{b}K}{r}, \qquad p_x = rK\tilde{p}_x, \qquad p_y = \frac{r^2}{a}\tilde{p}_y, \qquad c_x = r\tilde{c}_x, \qquad c_y = r\tilde{c}_y.$$

Remark 1. The system (3) above is called hybrid dynamical system [27], which is formulated by differential-difference-algebraic equations.