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An analytic solution of an oscillatory flow through a porous medium with radiation effect

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ABSTRACT

Analytical solutions for two-dimensional oscillatory flow on free convective-radiation of an incompressible viscous fluid, through a highly porous medium bounded by an infinite vertical plate are reported. The Rosseland diffusion approximation is used to describe the radiation heat flux in the energy equation. The resulting non-linear partial differential equations were transformed into a set of ordinary differential equations using two-term series. The dimensionless governing equations for this investigation are solved analytically using two-term harmonic and non-harmonic functions. The free-stream velocity of the fluid vibrates about a mean constant value and the surface absorbs the fluid with constant velocity. Expressions for the velocity and the temperature are obtained. To know the physics of the problem analytical results are discussed with the help of graph.

1. Introduction

Convective flows in a porous medium have received much attention in recent years due to wide applications in geothermal and oil reservoir engineering, as well as other geophysical and astrophysical studies, for example Rudraiah [1]. Raptis et al. [2] have studied hydromagnetic free convection flow through a porous medium between two parallel plates. Unsteady free convection flow through a porous medium bounded by an infinite vertical plate has been analyzed by Raptis [3]. But in recent years, there is a great resurgence in the study of heat transfer in channel flow with oscillatory flows, owing to the important of such studies in designing and improving liquid metal heat exchangers and several other practical applications. Oscillatory hydromagnetic flow through a porous medium in the presence of free convection and mass transfer flow is analysed by Hassanien [4]. The effect of temperature dependant heat source on hydromagnetic free convection oscillatory flow of an electrically conducting incompressible viscous fluid through a porous medium has been analyzed by Gholizadeh [5]. Hiremath and Patial [6] studied the effect on the oscillatory flow of a couple stress fluids through a porous medium. Heat transfer to MHD oscillatory flow in a channel filled with porous medium has been studied by Makinde and Mhone [7].

Moreover, considerable interest has been shown in radiation interaction with convection for heat in the fluid. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly in free convection problems involving absorbing-emitting fluids. The unsteady fluid flow past a moving plate in the presence of free convection and radiation were studied by Raptis and Perdikis [8], Makinde [9]. All these studies have been confined to unsteady flow in a non-porous medium. From the previous literature survey we observed that few papers were done in porous medium with oscillatory flow. Oscillatory flow through a porous medium has been studied by Raptis and Perdikis [10]. Keeping this in mind in the present analysis, an analytical solution for nonlinear oscillatory flow past a porous medium with radiation effect is investigated. The dimensionless governing equations for this investigation are solved analytically using two-term harmonic and non-harmonic functions.

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2. Mathematical analysis

Consider a two-dimensional, unsteady free convection with thermal radiation flow of a viscous, incompressible and electrically conducting flow through a highly porous medium which is bounded by a vertical infinite plane surface. The fluid is assumed to be a gray, absorbing-emitting but non-scattering medium. We assume that the surface absorbs the fluid with a constant velocity and the fluid far away from the surface vibrates about a mean value with direction parallel to the x' axis. All the fluid properties are assumed constant except the influence of the density variation with temperature which is considered only in the body force term. Also, the influence of the density variation in the other terms of the momentum and energy equations and the variation of the expansion coefficients with temperature is negligible. Under these assumptions, the governing nonlinear equations expressing the conservation of mass, momentum and energy are

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\rho\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = -\rho g + \mu\frac{\partial^2 u'}{\partial y'^2} - \frac{\mu}{k'}u' \tag{2}$$

$$\rho\frac{\partial v'}{\partial t'} = -\frac{\partial p}{\partial y'} - \frac{\mu}{k'}v' \tag{3}$$

$$\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'} = \alpha\left(\frac{\partial T^2}{\partial y'^2} - \frac{1}{k_0}\frac{\partial q^r}{\partial y'}\right). \tag{4}$$

The radiative heat flux term is simplified by using the Rosseland approximation [11] as

$$q^r = \frac{-4\sigma}{3k_1}\frac{\partial T^4}{\partial y'}. \tag{5}$$

The appropriate boundary conditions for velocity and temperature fields are

$$\begin{aligned} &u' = 0; \qquad T = T_W; \qquad y' = 0 \\ &u' = U'(t') \to U'(1 + e^{i\omega t'}); \qquad T = T_\infty; \qquad y' \to \infty \end{aligned} \tag{6}$$

where u' and v' being the components of the velocity corresponding to x' and y' axes respectively, ρ the density of the fluid, P the pressure, g the acceleration due to gravity, μ the viscosity, k' the permeability of the porous medium, T the temperature of the fluid, T_w the temperature of the surface, T_∞ the temperature of the fluid far away from the surface, k is the thermal conductivity of the fluid, U' a constant velocity, ω the frequency of vibration of the fluid, q^r the radiative heat flux in the y' direction, σ Stefan–Boltzmann constant and k_1 the mean absorption coefficient. The radiative heat flux in the x' direction is considered negligible in comparison with that in the y' direction [11]. The x' axis is taken along the plane surface with the direction opposite to the direction of gravity and y'-axis is taken to be normal to the surface. The physical variables are functions of y' and the time t' are described in [4].

Since the surface absorbs the fluid with a constant velocity, the continuity equation (1) gives that

$$v' = -v_0 = \text{constant}.$$

Using the free-stream and eliminating the pressure by using the constitutive equation [10]. In this view Eqs. (2)–(4) are reduce to

$$\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'} = g\beta(T - T_\infty) + v\frac{\partial^2 u'}{\partial y'^2} - \frac{v}{k'}u' \tag{7}$$

$$\frac{\partial T}{\partial t'} - v_0\frac{\partial T}{\partial y'} = \alpha\left(\frac{\partial T^2}{\partial y'^2} - \frac{1}{k_0}\frac{\partial q^r}{\partial y'}\right). \tag{8}$$

Introducing the following dimensionless quantities

$$\begin{aligned} &y = \frac{v_0\, y'}{v}; \qquad t = \frac{v_0^2\, t'}{v}; \qquad w = \frac{hv\, w'}{v_0^2}; \qquad u = \frac{u'}{U_0}; \\ &\theta = \frac{(T - T_\infty)}{T_W - T_\infty}; \qquad k = \frac{v_0^2\, k'}{v^2}. \end{aligned} \tag{9}$$

In view of the above non-dimensional variables, Eqs. (7) and (8) are reduced to non-dimensional form as

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - \frac{1}{k}u \tag{10}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{pr}\left[1 + \frac{4}{3}R(\theta + \phi)^3\right]\frac{\partial^2\theta}{\partial y^2} + \frac{4R}{pr}(\theta + \phi)^2\left(\frac{\partial \theta}{\partial y}\right)^2. \tag{11}$$