Available online at www.sciencedirect.com
ScienceDirect
Nonlinear Analysis: Hybrid Systems 2 (2008) 735–749
Nonlinear Analysis: Hybrid Systems
www.elsevier.com/locate/nahs

# Online clustering of switching models based on a subspace framework

K.M. Pekpe[a,1], S. Lecœuche[b,*]

[a] LAGIS - UMR CNRS 8146, Universite des Sciences et Technologies de Lille U.S.T.L., Cite Scientifique 59655 Villeneuve d'Ascq Cedex, France
[b] LAGIS - UMR CNRS 8146, Ecole des Mines de Douai, Département Informatique et Automatique, 59508 Douai Cedex, France

**Abstract**

This paper deals with the modelling of switching systems and focuses on the characterization of the local functioning modes using the online clustering approach. The system considered is represented as a weighted sum of local linear models where each model could have its own structure. This implies that the parameters and the order of the switching system could change when the system switches. Moreover, possible constants of the local models are also unknown. The method presented consists of two steps. First, an online estimation method of the Markov parameters matrix of the local linear models is established. Secondly, the labelling of these parameters is done using a dynamical decision space worked out with learning techniques; each local model being represented by a cluster. The paper ends with an example and a discussion with an aim of illustrating the method's performance.


*Keywords:* Identification; Markov parameters estimation; Machine learning; Dynamical classification; Switching systems

## 1. Introduction

Switching systems are a particular class of hybrid systems which can be considered as a weighted sum of the linear local models with normalized weights. In this type of modelling, contrary to the more general hybrid systems, only one model is active at each time and then the weights are binary. In fact, a great number of complex industrial applications which include both discrete-events and continuous phenomena could be modelled by switching systems, these latter applications having interesting approximation properties. The identification of such systems is a recent and active research area of increasing activity. A large majority of the contributions published in the hybrid identification literature deals with the subclass of piecewise affine models and several methods have been proposed. The challenge of the identification of hybrid systems could be summarized as the need to distinguish the current local mode and at the same time to estimate the model parameters. Then, each hybrid identification technique is a combination of clustering, regression, and optimization. A clustering method based on hyperplane determination is proposed in [6] which consists of the estimation of two hyperplanes continuously joined together. Inspired by pattern recognition

* Corresponding author. Tel.: +33 0 327712445; fax: +33 0 327712980.
*E-mail addresses:* midzodzi.pekpe@univ-lille1.fr (K.M. Pekpe), lecoeuche@ensm-douai.fr (S. Lecœuche).
[1] Tel.: +33 0 320434565; fax: +33 0 320337189.

1751-570X/$ - see front matter 
doi:10.1016/j.nahs.2007.11.006

techniques, [10] proposes to partition the regressor space into regions on which a linear local model is valid and then, estimate each model's parameters by standard least squares regression. The method proposed in [4] uses a mixed integer programming technique which is NP-hard in the worst case and is unfortunately practically applicable only when the number of data is very small. An extension of this method has been presented in [21] where the solution is of a lower complexity, thanks to the introduction of a change detection algorithm. This algorithm requires the use of a sliding window where only one switch is allowed; this implies an assumption on the minimal duration between the two switches. In the above methods, the order of the system cannot change. [24] proposes an identification of pwarx hybrid models with unknown and different orders. This work is based on an algebraic approach in which homogeneous polynomials are used to realize a segmentation of the regression space into regions which correspond to discrete states. This approach is ideal only in a noise-free context (see also [14]). It is also possible to cluster (in a suboptimal way) the generated data by sorting them using Bayesian inference, but again, the system order and the number of modes have to be *a priori* known [13]. Some common characteristics of these previous methods are that they are iterative but difficult to apply on-line. Most of them rely on data partitioning either to compute the regions of the regression space and then the related parameters [13,10,21] or to estimate first the parameters so as to derive the switching times [24].

A majority of works deal with the identification of ARX models such as switched ARX models and piecewise affine models. Contrary to input–ouput models, state-space representations are more appropriate to systems analysis in the case of multivariable models. Some authors [18,5] have recently addressed the problem of subspace identification in switching MIMO systems in an off-line context. In [22], the identification problem of piecewise linear systems under a state-space model is undertaken by using the assumptions that the number of modes and the switching times are known. The state representation of local models is determined by subspace methods. To obtain a complete representation of the piecewise linear system, transformations between the linear local models are used to characterize the models into the same basis. In the context of possible structure changes, [19] uses a change detection technique to estimate the switches and a subspace method to estimate the Markov parameters (state representation) of the local models. In this case, as the state is generally unknown, the partitioning of the regression space becomes harder. The detection of the switching times seems to be the alternative to deal with switching MIMO systems using subspace methods but the subspace approach requires a minimum residence time for each mode.

In fact, switching systems may also be regarded as time-varying systems. In this case, the data used off-line for the identification of such systems may cover only some modes of the systems and not the whole. The lack of adaptation abilities is probably the weak point of off-line identification methods. This is mainly due to the fact that online mode clustering does not seem to have received great attention in the hybrid systems community. To the best of our knowledge, only [23,11,3] deal with the identification of switching systems in an online context. The papers proposed by R. Vidal are a clever extension of his algebraic geometric approach in order to enable their application without knowledge of the number of modes and their order, but are limited to SISO systems. L. Bako's paper presents a recursive technique based on the state-space model and subspace tracking algorithms and demonstrates the possibility of identifying online MIMO linear piecewise systems in a recursive way using a detection approach. Recently, [16] presents an approach based on the combination of a recursive identification technique and a classifier set for non-stationary environments. But, this approach is limited to non-stationary systems with a fixed structure.

From this concise overview (see [20,17] for a precise overview), we note that few works have been dedicated to online identification of hybrid systems and no work has been addressed in the context of state-space modelling without constraints on the order and on the number of models. In this paper, we propose an online method for switching system modelling based on state-space models. The proposed technique is complementary to the other existing ones in the sense that it could be applied to a new range of identification problems. This method is designed to be recursively applied to MIMO hybrid systems with minimal *a priori* knowledge. The only required assumptions are: each model has to be stable, and the period before two switches should be greater than a lower bound. The number of the local linear models is unknown and could change with time. The order could change for each local model and constants of the local linear models could occur.[2] In fact, the knowledge of the system is obtained using online estimation and unsupervised continuous learning. The approach developed here is similar to the one proposed in [16]: one stage for the estimation and one stage for the classification.

[2] Constants could correspond to set points and/or constant faults on sensors or actuators.