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Optimal mode-switching for hybrid systems with varying initial states[☆]

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Abstract

This paper concerns a particular aspect of the optimal control problem for switched systems that change modes whenever the state intersects certain switching surfaces. These surfaces are assumed to be parameterized by a finite dimensional switching parameter, and the optimization problem we consider is that of minimizing a given cost-functional with respect to the switching parameter under the assumption that the initial state of the system is not a priori known. We approach this problem from two different vantage points by first minimizing the worst possible cost over the given set of initial states using results from min–max optimization. The second approach is based on a sensitivity analysis in which variational arguments give the derivative of the switching parameters with respect to the initial conditions.

1. Introduction

Over the last couple of decades, a lot of effort has been directed towards optimal control of hybrid systems, e.g. [9, 5,16–18,24,12,2]. Hybrid systems are complex systems that are characterized by discrete logical decision making at the highest level and continuous variable dynamics at the lowest level. Examples where such systems arise include situations where a control module has to switch its attention among a number of subsystems [19,21,23], or collect data sequentially from a number of sensory sources [10,13,18].

The type of hybrid systems under consideration in this paper belongs to the class of *switched autonomous systems*, where the continuous-time control variable is absent and the continuous-time dynamics change at discrete times (*switching-times*). For these, it is possible to derive gradient expressions for the cost functional with respect to the switching times when the initial state is fixed. In particular, [14] presented a gradient-based algorithm that finds optimal switching-times, dictating when to switch between a given set of modes, for the case when the switching-times are controlled directly. Furthermore, [6] considered the case when a switch between two different modes occurs when the state trajectory intersects a switching surface, defined by $g(x(t), a) = 0$, where $x(t) \in \mathbb{R}^n$ is the state of

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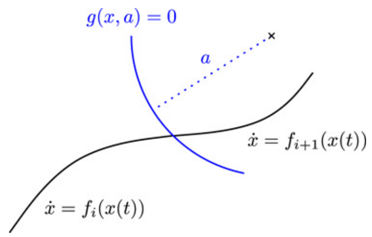

Fig. 1. Mode switching occurs when the state trajectory intersects a switching surface. In this case, the switching surface is a circle parameterized by the radius a.

the system at time t, and a parameterizes the switching surface. Reference [6] can thus be thought of as the starting point for this paper, as we consider a similar problem, but instead of optimizing with respect to a given fixed initial condition $x_0 \in \mathbb{R}^n$, we will assume that the initial state can be anywhere within a given set $S \subset \mathbb{R}^n$. This problem arose, for example, in the context of a recent DARPA sponsored robotics competition (LAGR — Learning Applied to Ground Robots), where an autonomous mobile robot was to navigate an unknown, outdoor environment from an initial set (the robot could start anywhere in the set) to a given target destination [22].

In order to find a good value of the switching parameter a, independent of the starting point from S, we will use the gradient formula presented in [6] and find a locally optimal a such that we will minimize the worst possible cost for all trajectories starting in S. Hence, we have a min–max problem and the results presented in this paper are based on the initial study found in [4].

An alternative view, initiated in [8], that will also be pursued in this paper is to assume that a switching surface can be obtained by varying the initial conditions and then solving for the corresponding, varying optimal switching times (and consequently switching states). In this manner, a sensitivity-based approach can be exploited for obtaining suitable switching surfaces.

The outline of this paper is as follows: In Section 2, the problem at hand is introduced together with some previous results relating to the gradient formula. Section 3 presents our solution using a min–max strategy. This is followed by a sensitivity analysis in Section 4 together with a discussion about the transition from sensitivities to switching surfaces. The conclusions are given in Section 5.

2. Switching parameter optimization

The type of systems under consideration in this paper are of the form

$$\dot{x}(t) = f_i(x(t)), \quad t \in [\tau_{i-1}, \tau_i),\ i \in \{1, \ldots, N+1\},\ x \in \mathbb{R}^n, \tag{1}$$

where we assume that the system switches N times between $N+1$ different dynamic regimes (or modal functions) at times $\tau_i, i = 1, \ldots, N$ over the time window $[0, T]$. (In the formulation above, we assume that $\tau_0 = 0$ and $\tau_{N+1} = T$, i.e. the final time.) Moreover, we assume that the switching times are not controlled directly. Instead, a switch occurs whenever the state trajectory intersects a switching surface and we assume that the geometry and dynamics of the system are such that the system does in fact undergo exactly N switches on the interval $[0, T]$ and that the intersections of the switching surfaces occur in a non-tangential manner. This problem was initially considered in [7] for a fixed initial state.

In this paper, we assume that the surfaces are defined by the solutions of parameterized equations from $\mathbb{R}^n$ to $\mathbb{R}$. We denote this parameter by a and suppose that $a \in \mathbb{R}^k$ for some integer $k \geq 1$, as shown in Fig. 1. For every switching surface g_j (denoting the surface that corresponds to the j^{th} switch), we let $g_j : \mathbb{R}^n \times \mathbb{R}^k \to \mathbb{R}$ be a continuously differentiable function. For a given fixed value of $a \in \mathbb{R}^k$, denoted here by a_j, we thus let the switching surface be defined by the solution points x of the equation $g_j(x, a_j) = 0$ as illustrated in Fig. 1. (Note that under mild assumption, the switching surface is a smooth $(n-1)$ dimensional manifold in $\mathbb{R}^n$, and a_j can be viewed as a control parameter of the surface.)

In order to minimize a cost function of the form

$$J = \int_0^T L(x(t))\mathrm{d}t, \tag{2}$$