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Fuzzy physical programming for Space Manoeuvre Vehicles trajectory optimization based on hp-adaptive pseudospectral method

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ABSTRACT

In this paper, a fuzzy physical programming (FPP) method has been introduced for solving multi-objective Space Manoeuvre Vehicles (SMV) skip trajectory optimization problem based on hp-adaptive pseudospectral methods. The dynamic model of SMV is elaborated and then, by employing hp-adaptive pseudospectral methods, the problem has been transformed to nonlinear programming (NLP) problem. According to the mission requirements, the solutions were calculated for each single-objective scenario. To get a compromised solution for each target, the fuzzy physical programming (FPP) model is proposed. The preference function is established with considering the fuzzy factor of the system such that a proper compromised trajectory can be acquired. In addition, the NSGA-II is tested to obtain the Pareto-optimal solution set and verify the Pareto optimality of the FPP solution. Simulation results indicate that the proposed method is effective and feasible in terms of dealing with the multi-objective skip trajectory optimization for the SMV.

1. Introduction

Over the past couple of decades, trajectory optimization problems in terms of reentry vehicle [1–6] have attracted significant attention. One of the current objectives is the development of Space Manoeuvre Vehicles (SMV) for a dynamic mission profile. The Mach number and the flight altitude of the reentry vehicle vary largely during the whole reentry phase, the aerodynamic feature of the vehicle has large uncertainties and nonlinearities. Due to these reasons, the use of numerical methods to handle these types of problems is commonly used. Numerical methods for solving optimal control problems are divided into two major classes: indirect methods and direct methods [7–10]. However, it is very difficult to solve the trajectory design problem by using indirect methods based on maximum principle. Therefore, direct optimization method has been widely used for trajectory optimization. Applying direct methods meant the development of several discrete methods [11].

In recent years, collocation methods for transforming optimal control problems have increased in popularity [12,13]. There are two main kinds of collocation methods, local collocation method such as the direct collocation and global collocation method e.g. the pseudospectral [14–16]. In a pseudospectral method, the collocation points are based on quadrature rules and the basis function are Lagrange or Chebyshev polynomials. In contrast to the direct collocation method, pseudospectral method usually divides the whole time history into a single mesh interval whereas its counterpart, direct collocation, divides time interval into several equal step subintervals and the convergence is achieved by adding the degree of the polynomial. To improve accuracy and computational efficiency using pseudospectral method, Darby presented a hp-strategy in [17–19]. By adding collocation points in a certain mesh interval or dividing the current mesh into subintervals simultaneously, the accuracy of interpolation can be improved dramatically.

Generally, the traditional trajectory design usually aims at one single objective, for example, minimizing the aerodynamic heating, maximizing the cross range, etc. However, in reality, for space vehicle trajectory design, most the missions contain more than one requirements and this brings the development of multi-objective optimization (MOO) [20]. There are many multi-objective methods which are suitable for these kind of problems. Commonly, the method based on weighting factors is widely used to transform different criteria into only one single objective but it is difficult to determine the weight coefficients. In 1996, Messac designed a physical programming (PP) method to convert the objectives [21,22], which removes the information of priority and weight coefficients. But in practice, usually there are some fuzzy factors in the real system and because of this, a fuzzy physical programming method is proposed in this paper.

The mission scenario investigated in this paper focuses on the atmospheric skip hopping, targeting the entry into the atmosphere

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down to a predetermined position (predetermined altitude given by the industrial sponsor of this project) and the required controls involved in returning back to low earth orbit. Studies can be found in the literature regarding the skip reentry of deep-space spacecraft with high speed over first cosmic velocity, however in the scenario considering in this paper, a high thrust engine would be necessary for SMV to return to low earth orbit. The overall mission can be found in Fig. 1. General skip reentry can be divided into five phases: initial roll, down control, up control, Kepler and final entry. Considering the mission of the SMV is to overfly the ground target with specific altitude, the most challenging phases 2 and 3 will be considered in this paper.

Most of the current studies in trajectory optimization are based on the numerical simulation. Smirnov et al. [23,24] presented studies in terms of developing mathematical model for evaluation of stochastic numerical errors accumulation. Based on the published simulation results, the problem of accumulation of errors cannot be ignored. Therefore, the effect of noise on the trajectory optimization is also considered in this work, and the results are presented in Section 5 of this paper.

The paper is organized as follows. In Section 2, introduce the aerodynamic model of the SMV reentry vehicle and some basic principles of the trajectory optimization problem. Section 3 describes the method used to discretize the optimal control problem. Then in Section 4 the fuzzy physical programming (FPP) procedures of solving multi-objective SMV trajectory problem is detailed. Following that, Section 5 presents comparison between solution calculated for each single-objective and the compromised solution generated by employing the FPP approach.

2. Problem description

2.1. SMV dynamic model

The earth is considered as a symmetrical sphere and the earth rotation is ignored. Considering a three degree of freedom dynamic equations of SMV reentry vehicle:

$$\begin{aligned}
\dot{r} &= V \sin\gamma \\
\dot{\theta} &= \frac{V \cos\gamma \sin\psi}{r \cos\phi} \\
\dot{\phi} &= \frac{V \cos\gamma \cos\psi}{r} \\
\dot{V} &= \frac{T \cos\alpha - D}{m} - g \sin\gamma \\
\dot{\gamma} &= \frac{L \cos\sigma + T \sin\alpha}{mV} + \left(\frac{V^2 - gr}{rV}\right)\cos\gamma \\
\dot{\psi} &= \frac{L \sin\sigma}{mV \cos\gamma} + \frac{V}{r}\cos\gamma \sin\psi \tan\phi \\
\dot{m} &= -\frac{T}{I_{sp} g} \\
\dot{\alpha} &= K_\alpha(\alpha_c - \alpha) \\
\dot{\sigma} &= K_\sigma(\sigma_c - \sigma) \\
\dot{T} &= K_T(T_c - T)
\end{aligned} \tag{1}$$

where r is the radial distance from the Earth center to the vehicle, θ and ϕ are the longitude and latitude, V is the Earth-relative velocity, γ is the relative flight-path angle, ψ is the relative velocity heading angle measured clockwise from the north, m is the mass of the vehicle, t is time, control variables are angle of attack α_C, bank angle σ_C and thrust T_C, respectively. In reality, the real control variables cannot change dramatically (i.e. from lower bound to upper bound). Therefore, in the model provided (1), three rate constraints are introduced by using the technique of first order lag which can be concluded to the last three equations in (1).

The atmosphere model, lift L and drag D can be defined as:

$$\begin{aligned}
g &= \frac{\mu}{r^2} \qquad \rho = \rho_0 \exp\frac{r - r_0}{h_s} \\
L &= \frac{1}{2}\rho V^2 C_L S \quad D = \frac{1}{2}\rho V^2 C_D S
\end{aligned} \tag{2}$$

where $S = 2690\ \mathrm{ft}^2$ is the reference area, $\mu = 1.4076539 \times 10^{16}\ \mathrm{ft}^3/\mathrm{s}^2$ is the gravitational parameter of the earth. ρ is the density of atmosphere and $\rho_0 = 0.002378\ \mathrm{slug/ft}^3$ is the density of atmosphere at sea-level. $r_0 = 20,902,900$ ft is the Earth's radius, C_L and C_D are lift and drag coefficient determined by angle of attack α and Ma, respectively, g is the gravity acceleration.

The drag and lift coefficient can be determined by the following equations:

$$\begin{aligned}
C_D &= C_{D0} + C_{D1}\alpha + C_{D2}\alpha^2 \\
C_L &= C_{L0} + C_{L1}\alpha
\end{aligned} \tag{3}$$

where

$C_{L0} = -0.2070$, $C_{L1} = 1.676$, $C_{D0} = 0.07854$, $C_{D1} = -0.3529$, $C_{D2} = 2.040$

2.2. Reentry process constraints

SMV reentry process should satisfy some constraints due to safety reasons and also depending on the mission requirements. These constraints can be summarized as initial and terminal constraints, path constraints and boundary constraints.

2.2.1. Initial and terminal constraints

The complete mission can be divided into two phases, the descent phase and exit phase. Due to the mission requirement, the state variables at minimum decent point are specified. The initial conditions of all the states are:

$$[r, \phi, \theta, V, \gamma, \psi, m, \alpha, \sigma, T] = [r_0, \phi_0, \theta_0, V_0, \gamma_0, \psi_0, m_0, \alpha_0, \sigma_0, T_0] \tag{4}$$

On the other hand, at the minimum altitude point and final point (i.e. final point to return back into low earth orbit), hence complete one hop, the terminal altitude constraints are:

$$[r(1), r(f)] = [r_b, r_f] \tag{5}$$

where $r(1)$ and $r(f)$ stand for altitude value at bottom point and final time, respectively.

2.2.2. Path constraints

During the whole time period, to protect the structure of reentry vehicle, in simulation the SMV model needs to satisfy strict path constraint, which can be summarized as follows:

$$\begin{aligned}
\dot{Q}_d &= K_Q \rho^{0.5} V^{3.07}(c_0 + c_1\alpha + c_2\alpha^2 + c_3\alpha^3) < \dot{Q}_{dmax} \\
P_d &= \frac{1}{2}\rho V^2 < P_{dmax} \\
n_L &= \frac{\sqrt{L^2 + D^2}}{mg} < n_{Lmax}
\end{aligned} \tag{6}$$

where $c_0 = 1.067$, $c_1 = -1.101$, $c_2 = 0.6988$, $c_3 = -0.1903$ and $K_Q = 9.289 \times 10^{-9}\ \mathrm{Btu\ s^{2.07}/ft^{3.57}/slug^{0.5}}$. Q_{dmax}, P_{dmax} and n_{Lmax} represents allowable maximum heating rate, dynamic pressure and acceleration, respectively.

2.2.3. Boundary constraints

For the SMV, the states should be limited as: