

# Vibration suppression of composite laminated plate with nonlinear energy sink

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## ABSTRACT

The composite laminated plate is widely used in supersonic aircraft. So, there are many researches about the vibration suppression of composite laminated plate. In this paper, nonlinear energy sink (NES) as an effective method to suppress vibration is studied. The coupled partial differential governing equations of the composite laminated plate with the nonlinear energy sink (NES) are established by using the Hamilton principle. The fourth-order Galerkin discrete method is used to truncate the partial differential equations, which are solved by numerical integration method. Meanwhile study about the precise effectiveness of the nonlinear energy sink (NES) by discussing the different installation location of the nonlinear energy sink (NES) at the same speed. The results indicate that the nonlinear energy sink (NES) can significantly suppress the severe vibration of the composite laminated plate with speed wind loadings in to protect the composite laminated plate from excessive vibration.

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## 1. Introduction

Composite laminated plate has been more and more widely used in aerospace field (e.g.: wing, steam, plate, etc.), because of the light weight and high strength/stiffness-to-weight ratio. However, in the airflow, due to the existence of aerodynamic force, the excessive vibration of the plate is very common. In order to solve this problem, various researchers had studied about the plate vibration. The deliberate survey of the plate vibration was given by Dowell [1,2] and Liao [3]. Lee et al. [4] and Oh et al. [5] studied about the linear and nonlinear vibration of the stiffened panel with aerodynamic pressure using the finite element method. In order to reduce the possibility of flight accidents caused by the excessive vibration, for this reason many researchers studied about how to suppress the vibration of the plate. Moon and Hwang [6] presented an optimal strategy to control the flutter of a composite panel with aerodynamic pressure using piezoelectric actuators. Song and Li [7] studied about the active vibration control of the composite laminated plate with aerodynamic pressure using piezoelectric actuator/sensor pairs. Li et al. [8] also studied the active vibration control using the active constrained layer damping treatment.

However, all of the aforementioned active suppression

methods need sensors and evaluator equipment, which are more complex than the passive ones. Instead, passive methods are inherently stable and simple to design, which need no additional energy and sensors. Thus they are more suitable to stabilize plates in engineering. Various researchers have carried out some studies on passive vibration control. Zhang et al. [9,10] studied the isolation platform of control moment gyroscopes. Many researchers have gotten some achievements and some applications of engineering field [11–14]. However, traditional linear vibration absorbers only have great respond at the natural frequency.

In this paper, nonlinear targeted energy transfer is studied. The nonlinear energy sink (NES) which is made up of a mass, a damper and a nonlinear spring was studied to suppress the vibration of the composite laminated plate. The nonlinear energy sink has a small additional mass and responds a broad frequency. Gendelman et al. [15] studied the nonlinear targeted energy transfer at first. The way has been proved that it works at an extensive frequency range. Works have been done in many structures, such as beam [16] rod [17] and plate [18]. Lee et al. [19] attached the NES to the fixed wing of the plane for vibration suppression of the limit cycle. Nucera et al. [20] used the NES to a multistory frame structure for vibration suppression. Mehmood et al. [21] investigated the effectiveness of a nonlinear energy sink (NES) on a vortex-induced vibration of a circular cylinder. Luongo and Zulli [22] studied nonlinear energy sink to control elastic strings. They obtained the beneficial effect of the NES to reduce the amplitude of the string. Luongo and Zulli [23,24] also applied a blended Multiple Scale/

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Harmonic Balance Method to investigate the dynamic analysis of an externally excited NES-controlled system and the aeroelastic instability analysis of an NES-controlled system. Vaurigaud et al. [25,26] proposed a new methodology using parallel nonlinear energy sinks for targeted energy transfer.

This paper focuses on the vibration suppression of the composite laminated plate with aerodynamic pressure. The Kirchhoff plate model, the first-order piston theory and the Hamilton principle are used to establish the motion equation of the plate. Using the fourth-order Galerkin discrete method is aimed to truncate the partial differential equations into a set of ordinary differential equations, which are then solved by Runge–Kutta numerical integration method. Meanwhile study about the effectiveness of NES by discussing the different installation location of NES at the same speed. The results indicate that the NES can significantly suppress the severe vibration of the composite laminated plate with speed wind loading to protect the plate from excessive vibration.

## 2. Equation of motion of plate and the nonlinear energy sink (NES)

In this paper the Kirchhoff plate model is used to investigate. As shown in the Fig. 1, the per layer length, width, thickness of the composite laminated plate are  $a$ ,  $b$ ,  $h$  respectively. The total thickness is  $h_n$ . The arbitrary point axial and transverse displacements in the composite laminated plate [7]:

$$u = -z \frac{\partial w}{\partial x}, \quad v = -z \frac{\partial w}{\partial y}, \quad w = w \quad (1)$$

where  $z$  is the co-ordinate in the transverse direction of the plate.

The strain-displacement relationship of the composite laminated plate can be expressed as Eq. (2),

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}^T = \begin{bmatrix} -z \frac{\partial^2 w}{\partial x^2} \\ -z \frac{\partial^2 w}{\partial y^2} \\ -2z \frac{\partial^2 w}{\partial x \partial y} \end{bmatrix}^T = z [\kappa_x, \kappa_y, \kappa_{xy}]^T = z [\kappa] \quad (2)$$

where  $\varepsilon_x$ ,  $\varepsilon_y$ ,  $\gamma_{xy}$  are the normal strains in  $x$ ,  $y$  direction and the shear strain in  $x$ – $y$  plane,  $\kappa_x$ ,  $\kappa_y$ ,  $\kappa_{xy}$  are the bending curvatures, respectively, and  $[\kappa]$  is the bending curvatures vector.

The constitutive equation of the  $k$ th layer of the composite laminated plate when transformed to the laminate general coordinate  $(x, y)$  can be expressed as Eq. (3),

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\bar{Q}]_{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{pmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ & \bar{Q}_{22} & \bar{Q}_{26} \\ \text{sym} & & \bar{Q}_{66} \end{pmatrix}_{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (3)$$

where  $\sigma_x$ ,  $\sigma_y$  and  $\tau_{xy}$  are the normal stress in  $x$  and  $y$  directions and shear stress in  $x$ – $y$  plane, respectively, is the elastic constants in the laminate general coordinate which depends on the lamina

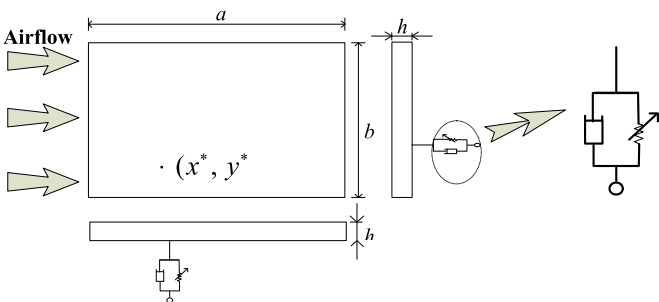


Fig. 1. The sketch of the composite laminated plate with the NES.

stiffness coefficients  $Q_{ab}$ .

The aerodynamic load  $\Delta p$  can be expressed by the supersonic piston theory as Eq. (4) by [7],

$$\begin{cases} \Delta p = -\xi \frac{\partial w}{\partial x} - \mu \frac{\partial w}{\partial t} \\ \xi = \rho_\infty U_\infty^2 / \sqrt{M_\infty^2 - 1} \\ \mu = \rho_\infty U_\infty (M_\infty^2 - 2) / (M_\infty^2 - 1)^{3/2} \end{cases} \quad (4)$$

where  $\xi$ ,  $\mu$ ,  $\rho_\infty$ ,  $U_\infty$ ,  $M_\infty$  are the aerodynamic pressure, damping parameters, the free stream air density, velocity and Mach number, respectively.

The NES is installed in the lee of the composite laminated plate in order to absorb vibration energy. The force between the NES and the plate can be described as the equations of the relative displacement and speed at the installation location. Therefore the force between the NES and the plate can be expressed as Eq. (5),

$$g(w, s) = k[s - w(x^*, y^*, -0.5h)]^3 + c[\dot{s} - \dot{w}(x^*, y^*, -0.5h)] \quad (5)$$

where  $s$  and  $\dot{s}$  are the displacements and the speed of the NES, respectively. The  $k$  and  $c$  are the stiffness factor and the damping of the NES respectively. The  $h$  is thickness of the composite laminated plate. The NES installation location is  $(x^*, y^*, -0.5h)$ .

The equation of motion of the NES can be expressed as Eq. (6),

$$m\ddot{s} = k \left[ \sum_{i=1}^4 c_i(t) \phi_i(x^*, y^*) - s \right]^3 + c \left[ \sum_{i=1}^4 c_i(t) \dot{\phi}_i(x^*, y^*) - \dot{s} \right] \quad (6)$$

The governing equations of motion of the composite laminated plate with the NES can be expressed Hamilton principle as Eq. (7),

$$\int_{t_0}^{t_1} (\delta T + \delta V - \delta U) dt = 0 \quad (7)$$

where  $\delta T$ ,  $\delta U$ ,  $\delta V$ , are the virtual kinetic energy, virtual potential energy and virtual work done by external load respectively.  $t_1 - t_0$  is the integration time.

$$\begin{cases} \delta T = \int_0^b \int_0^a \int_{-h/2}^{h/2} \rho_0 (u \delta u + v \delta v + w \delta w) dz dx dy \\ \delta U = \int_0^b \int_0^a \int_{-h/2}^{h/2} (\sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + 2\tau_{xy} \delta \gamma_{xy}) dz dx dy \\ \delta V = \int_0^b \int_0^a [g(w, s) \delta[(x - x^*)(y - y^*)] \\ \quad \times \delta w(x^*, y^*, -0.5h) - \Delta p \delta w(x, y, -0.5h)] dx dy \end{cases} \quad (8)$$

where  $\rho_0$  is the plate density,  $\delta$  is the Dirac delta function.

Substituting Eqs. (3)–(5) into Eq. (6) and setting the coefficients of virtual displacements,  $\delta T$ ,  $\delta U$ ,  $\delta V$  to zero yields.

$$\begin{cases} \frac{\partial^2 N_{xx}}{\partial x^2} + \frac{\partial^2 N_{yy}}{\partial y^2} + 2 \frac{\partial^2 N_{xy}}{\partial x \partial y} + g(w, s) \delta[(x - x^*)(y - y^*)] \\ \quad - \Delta p = I_0 \ddot{w} - I_1 \left( \frac{\partial^2 \dot{w}}{\partial x^2} + \frac{\partial^2 \dot{w}}{\partial y^2} \right) \\ I_0 = \int_{-h/2}^{h/2} \rho_0 dz \\ I_1 = \int_{-h/2}^{h/2} \rho_0 z^2 dz \end{cases} \quad (9)$$

where the  $I_0$ ,  $I_1$  is the moments of inertia.

The force resultants operator  $M_{kl}$  ( $k, l = x, y$ ) can be expressed as Eq. (10),

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