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A survey of different classes of Earth-to-Moon trajectories in the patched three-body approach☆

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ABSTRACT

This paper deals with Earth-to-Moon transfers in the patched three-body approach, in which the Sun–Earth–Moon–Spacecraft four-body system is approximated by two coupled Circular Restricted Three-Body Problems (CR3BP). This approach provides preliminary solutions that can be numerically refined into full four-body solutions. The standard transfers in this approach are low-energy manifold guided solutions with long transfer time which connect transit and non-transit orbits of each three-body system. Besides the standard transit-non-transit connections, there are alternative solutions involving a bi-parametric family of quasi-periodic orbits around the Earth. These solutions connect quasi-periodic orbits on two-dimensional tori of the Sun–Earth–Spacecraft system with L_1 or L_2 transit solutions of the Earth–Moon–Spacecraft system to provide transfers with lunar ballistic capture and short flight time. We review the dynamical elements employed to obtain the different classes of transfers and give examples of solutions obtained from sets of initial conditions around the Earth that are consistent with current infrastructure for space exploration.

1. Introduction

Lunar exploration has been receiving renewed attention, as attested by recent NASA's LADEE, ARTEMIS, and GRAIL missions, and CNSA's Chang'e 3 mission. As missions become more demanding, they often require alternative solutions based on N-body gravitational effects on the spacecraft, instead of the traditional orbits based on two-body conic solutions. These novel transfers allow us to reduce propellant mass or provide non-Keplerian orbits that fulfill specific mission requirements [1].

For example, in 2010, the two probes of the ARTEMIS mission navigated to the Earth–Moon Lagrangian points L_1 and L_2 and were kept in unstable quasi-periodic orbits to investigate how the Sun's radiation interacts with the Moon, and to survey the applicability of lunar regions as staging or communication relay locations [2]. Also, in 2011, the two spacecraft of the GRAIL mission used low-energy trajectories to leave the Earth, following the invariant structures associated to the Sun–Earth Lagrangian point L_1 to reach the Moon [3].

The trajectories used by these missions are solutions of the Circular Restricted Three-Body Problem (CR3BP).

Some restricted four-body systems can be modeled in an initial approach by two coupled three-body systems. This approximation, known as the *patched three-body approach*, provides preliminary solutions to be used as initial guess for a numerical procedure that converges to a full four-body solution [1,4]. In particular, the Sun–Earth–Moon–Spacecraft (Sc) system can be decomposed into the Sun–Earth–Sc (SE) and the Earth–Moon–Sc (EM) systems.

In fact, the rescue trajectory for the Hiten Mission, a paradigmatic example for a class of low-energy Earth-to-Moon orbits obtained by considering the gravitational effects of the Earth, the Moon, and the Sun on the motion of the spacecraft simultaneously [5–8], was later found to be related to the hyperbolic invariant manifolds of the CR3BP by employing the patched three-body approximation [9–11]. Besides trajectories connecting different hyperbolic objects, quasiperiodic orbits of the CR3BP have also been found to provide alternative transfer solutions in the Solar system. For example, there are different types of quasiperiodic orbits with long-term stability in the Mars–Phobos–Sc system that connect with the invariant manifolds of L_1 Halo orbits [12]. Quasi-periodic orbits were also studied as potential alternatives to long term monitoring of solar activity in the Sun–Earth system [13]. Moreover, conic or resonant arcs have been demonstrated to serve as transfer mechanisms between non-resonant orbits in N-body environments [14].

This paper presents a survey of preliminary Earth-to-Moon transfers in the patched three-body approach, using the planar

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case of the CR3BP. Essentially, two types of transfers with different dynamical properties are found by using this approach: (i) solutions which connect non-transit orbits associated to L_1 or L_2 of the SE system with transit orbits associated to L_2 of the EM system; and (ii) solutions which connect quasi-periodic orbits of the SE system with L_1 or L_2 transit orbits of the EM system.

The patching procedure is discussed and each transfer possibility is illustrated, giving concrete examples of solutions obtained from sets of initial conditions around the Earth that are consistent with current infrastructure for space exploration. Additionally, different ballistic capture scenarios are illustrated with some considerations on the general dynamics and the energy cost to reach them by means of quasi-periodic orbits of the SE system.

2. Mathematical model

The planar CR3BP describes the motion of a spacecraft moving in the gravitational field of two primaries P_1 and P_2, with masses m_1 and m_2 [15]. The equations of motion in the normalized synodic reference frame, are

$$\ddot{x} - 2\dot{y} = \Omega_x,$$
$$\ddot{y} + 2\dot{x} = \Omega_y, \tag{1}$$

with the effective potential given by

$$\Omega(x, y) = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{\mu(1-\mu)}{2}, \tag{2}$$

and

$$\mu = m_2/(m_1 + m_2), \quad m_1 > m_2,$$
$$r_1 = \sqrt{(x-\mu)^2 + y^2},$$
$$r_2 = \sqrt{(x+1-\mu)^2 + y^2}, \tag{3}$$

where r_1 and r_2 denote the distances from the particle to P_1 and P_2, respectively, and μ, known as the mass parameter of the CR3BP, is the dimensionless mass of P_2.

The normalized variables are such that the distance between P_1 and P_2, the sum of their masses, and their angular velocity around the barycenter are normalized to one. So, one complete rotation of the primaries around their barycenter with respect to an inertial frame occurs in 2π dimensionless units of time, and, in the synodic frame, P_1 and P_2 are fixed at $(\mu, 0)$ and $(\mu - 1, 0)$, respectively.

The planar CR3BP has a conserved quantity, the Jacobi constant:

$$J(x, y, \dot{x}, \dot{y}) = 2\Omega(x, y) - (\dot{x}^2 + \dot{y}^2) = C, \tag{4}$$

and five equilibrium points: L_1, L_2, and L_3, located on the x-axis, saddle-center type; L_4 and L_5, located at $(\mu - 1/2, \mp\sqrt{3}/2)$, linearly stable for $\mu \in (0, \mu_1)$, with $\mu_1 = (9 - \sqrt{69})/18$, where μ_1 is Routh's critical mass ratio. C_k denotes the Jacobi constant value at L_k, $k = 1, 2, 3, 4, 5$. For each μ, these values define the five possible Hill region configurations, corresponding to distinct transport possibilities through phase space.

For $C < C_k$, $k = 1, 2, 3$, there is a uniparametric family of retrograde periodic orbits around L_k, called Lyapunov orbits, denoted by Γ. The first elements of this family present Monodromy matrices with a pair of complex eigenvalues, ζ_1 and ζ_2, associated to the center manifold and a real pair of eigenvalues, ζ_3 and ζ_4, with $\zeta_3 < 1$, $\zeta_4 > 1$, and $\zeta_3\zeta_4 = 1$, associated to stable and unstable manifolds, respectively, defined by

$$W^s(\Gamma) = \{\mathbf{x} \in \mathbb{R}^4 : \phi(\mathbf{x}, t) \to \Gamma \text{ when } t \to \infty\},$$
$$W^u(\Gamma) = \{\mathbf{x} \in \mathbb{R}^4 : \phi(\mathbf{x}, t) \to \Gamma \text{ when } t \to -\infty\}. \tag{5}$$

W^s and W^u are locally homeomorphic to two-dimensional cylinders and act as separatrices in phase space for a given C, so that four categories of orbits are defined in the neck region around each L_k, $k = 1, 2, 3$: the Lyapunov orbit itself, orbits that approach it asymptotically, transit orbits that cross the neck region from one side to the other, and non-transit orbits [4,16,17]. For lower values of C, a bifurcation may occur and these periodic orbits of center-saddle type become saddle–saddle solutions.

3. The patched three-body approach

The patched three-body approach was introduced by Koon et al. to take advantage of the dynamical structure of the CR3BP to obtain preliminary solutions of restricted four-body systems described by the Concentric Circular Model and the Bicircular Model [1,9,10].

Indeed, in an initial approximation, the Sun–Earth–Moon–Sc can be decomposed into two coupled planar CR3BPs, given that the mean eccentricity of Moon's orbit around the Earth is approximately 0.05, the mean inclination to the ecliptic is 5.14°, and the mean eccentricity of Earth's orbit around the Sun is approximately 0.0167. The mass parameters of the SE and of the EM systems are, respectively, $\mu_{SE} = 3.03591 \times 10^{-6}$ and $\mu_{EM} = 1.21506683 \times 10^{-2}$. It is worth to mention that this value of μ_{SE} includes the mass of the Moon along with the mass of the Earth, so, in effect, the SE system corresponds to the Sun–(Earth–Moon) CR3BP.

In this paper, the coordinates in the SE synodic reference frame are denoted by $(X, Y, \dot{X}, \dot{Y})$, while $(x, y, \dot{x}, \dot{y})$ corresponds to the coordinates in the EM synodic reference frame.

For future reference, the critical values of the Jacobi constant in the first and the second Lagrangian points of the SE and the EM systems are given in Table 1.

The original key idea to design a preliminary Earth-to-Moon transfer in the patched three-body approach is to use two solution arcs to connect an initial geocentric orbit to a final selenocentric orbit: the first arc is a non-transit orbit associated to a Lyapunov orbit around $L_{1,2}$ of the SE system, and the second arc is a transit orbit associated to a Lyapunov orbit around L_2 of the EM system [1,10]. The two arcs connect because $W^u(\Gamma_{1,2}^{SE})$ and $W^s(\Gamma_2^{EM})$ intersect each other. Fig. 1(a) shows the projection onto position space in SE synodic coordinates of a two-piece transfer trajectory: the thick blue curve corresponds to the first arc and is a solution of the Sun–Earth–Sc CR3BP, while the black curve corresponds to the second arc and is a solution of the Earth–Moon–Sc CR3BP. Additionally, Fig. 1(b) shows the second arc projected onto position space in EM synodic coordinates.

The two pieces are connected at two states that share the same location in position space, but may have different velocities. To obtain a transfer solution it is useful to define a Poincaré section in the SE synodic frame where the involved manifolds intersect. For concreteness, consider the Poincaré section Σ_c defined by $X = -1 + \mu_{SE}$, $\dot{X} > 0$, $Y < 0$. Σ_c rotates around the Earth in the EM synodic frame, so at the patching instant, Σ_c corresponds to a section Σ_c^* in the EM system.

In Σ_c, the patching states are simultaneously outside the cut of

Table 1
Critical values of the Jacobi constant in the first and the second Lagrangian points of the SE and the EM systems.

C	Sun–Earth	Earth–Moon
C_1	3.0009000935260	3.2003449098321
C_2	3.0008960456047	3.0241502628815