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Invited Paper

Bounded relative orbits about asteroids for formation flying and applications

Nicola Baresi*, Daniel J. Scheeres*, Hanspeter Schaub*

Department of Aerospace Engineering Sciences, University of Colorado Boulder, 80309 Boulder, CO, United States

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The relative motion about 4179 Toutatis is studied in order to investigate the feasibility of formation flying as an alternative concept for future asteroid exploration missions. In particular, the existence of quasi-frozen orbits about slowly rotating bodies allows us to compute families of periodic orbits in the body-fixed frame of the asteroid. Since these periodic orbits are of the *center* $\times$ *center* type, quasi-periodic invariant tori are calculated via fully numerical procedures and used to initialize spacecraft formations about the central body. Numerical simulations show that the resulting in-plane and out-of-plane relative trajectories remain bounded over long time spans; i.e., more than 30 days.

1. Introduction

In this thriving era for small bodies exploration, it may be interesting to study the relative motion of satellites flying in a formation about asteroids or comets. Such a concept is not novel and has already been proposed as a potential benefit for several asteroid mitigation strategies. For instance, Maddock and Vasile considered formations of solar concentrators to deflect hazardous Near Earth Asteroids (NEA) by surface ablation [1]. Alternatively, Gong et al. proved the reliability of solar-sail formations in displaced orbits as effective and powerful gravity tractors [2].

A common denominator in the literature, however, is that the gravitational attraction of the asteroid is usually neglected or oversimplified. Both Gong [2] and Vasile [3] approximate the gravitational pull excerted by the central body via a point-mass gravity field. Only recently, Foster et al. considered multiple gravity tractors in a high-order spherical harmonics gravity field, but instead of designing cost-free relative trajectories, the authors were focused on controlling the satellites at fixed locations with respect to the Sun-asteroid rotating frame to maximize the effects of their proposed deflection strategy [4]. Accordingly, passive relative orbits in the proximity of small bodies are yet to be found and described.

In this paper, a systematic approach to establish bounded relative motion about slowly rotating tri-axial ellipsoids is presented. As a case study, a chief and a deputy spacecraft are considered while flying in a formation about 4179 Toutatis, a slowly rotating asteroid that was flown-by China's Chang'e 2 spacecraft in December 2012 [5]. Because of the existence of quasi-frozen orbits in the body-fixed frame of the asteroid [6], the secular evolution of the mean orbit elements of the satellites can be studied via Lagrange Planetary Equations [7]. Moreover, first-order bounded relative motion conditions can be derived by matching the averaged drift rates due to the nonspherical shape of the central body [8]. As these bounded relative motion conditions are based on mean orbit element differences, the applicability of using a first-order mean-to-osculating orbit element mapping for spacecraft formations about Toutatis is also investigated and used to motivate additional numerical analyses. Specifically, stable periodic orbits are computed starting

* Corresponding authors.
E-mail addresses: nicola.baresi@colorado.edu (N. Baresi), scheeres@colorado.edu (D.J. Scheeres), hanspeter.schaub@colorado.edu (H. Schaub).

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from the output of the mean-to-osculating orbit element mapping and using a Poincaré map between consecutive surface of section crossings [9]. Then, Kolemen's method is applied to extend the center submanifolds beyond the linear regime, and to compute quasi-periodic orbits that foliate two dimensional invariant tori in the neighborhood of the original periodic orbit [10]. Finally, the relative motion between satellites initialized on the quasi-periodic tori as well as on the computed periodic orbit is studied. In particular, numerical simulations investigate the long-term behavior of the relative motion and assess the robustness of the derived initial conditions while taking into account unmodeled forces such as solar radiation pressure and third body attraction.

2. Bounded relative orbit conditions

According to Reference [9], the majority of the perturbations felt by mass particles about asteroids are due to the second degree and order gravity field. Thus, for preliminary Formation Flying mission analyses, it is possible to consider the gravitational potential as given by

$$U=\frac{\mu}{r}+R=\frac{\mu}{r}\left\{1+\left(\frac{r_0}{r}\right)^2\left[C_{20}\left(\frac{3}{2}\sin^2\delta-\frac{1}{2}\right)\right.\right.$$
$$\left.\left.-3C_{22}\cos(2\lambda)\left(\sin^2\delta-1\right)\right]\right\}, \tag{1}$$

where μ is the gravitational parameter of the central body, r is the distance of the satellite from the center of the asteroid, δ and λ are its latitude and longitude with respect to the first principal axis, respectively, r_0 is the scale factor, and $C_{20}=-J_2$ and C_{22} are respectively the second zonal and second-degree second-order spherical harmonics coefficients.

Assuming that the asteroid is rotating about its maximum axis of inertia, one can rewrite δ and λ via

$$\sin\delta=\sin i\sin u, \tag{2a}$$

$$\tan\lambda=\frac{\sin\Omega_R\cos u+\cos\Omega_R\sin u\cos i}{\cos\Omega_R\cos u-\sin\Omega_R\sin u\cos i}, \tag{2b}$$

where i is the inclination of the spacecraft, $u=\omega+f$ is the argument of latitude, $\Omega_R=\Omega-\omega_T t$ is the longitude of the ascending node with respect to the rotating body-fixed frame of the asteroid, and ω_T is the spin rate of the central body. Furthermore, if the mean motion of the spacecraft is much larger than ω_T, all of the orbit elements can be regarded as constant over one orbit period [6]. Then, it is also possible to consider the averaged perturbing function over the mean anomaly M, i.e.,

$$\overline{R}=\frac{1}{2\pi}\int_0^{2\pi}R\,\mathrm{d}M=\frac{\mu r_0^2}{2a^3(1-e^2)^{3/2}}\left[C_{20}\left(\frac{3}{2}\sin^2 i-1\right)\right.$$
$$\left.-3C_{22}\sin^2 i\cos(2\Omega_R)\right] \tag{3}$$

and investigate the evolution of the spacecraft mean orbit elements with Lagrange Planetary Equations [7].

It turns out that for a very slow rotator such as 4179 Toutatis ($P_T=2\pi/\omega_T\simeq 5.43$ days), the mean orbital element rates can be rewritten as [6,8]

$$a'=0, \tag{4a}$$

$$e'=0, \tag{4b}$$

$$i'=\frac{3C_{22}\sin i\sin(2\Omega_R)}{\eta^4L^7}, \tag{4c}$$

$$\Omega_R'=\frac{3\cos i(C_{20}+2C_{22}\cos(2\Omega_R))}{2\eta^4L^7}-\frac{\omega_T}{n_0}, \tag{4d}$$

$$\omega'=-\frac{15\cos(2i)(C_{20}+2C_{22}\cos(2\Omega_R))+9C_{20}-6C_{22}\cos(2\Omega_R)}{8\eta^4L^7}, \tag{4e}$$

$$M'=\frac{1}{L^3}+\frac{9\sin^2 i(C_{20}+2C_{22}\cos(2\Omega_R))-6C_{20}}{4\eta^3L^7}, \tag{4f}$$

where

$$(\cdot)'=\frac{1}{n_0}\frac{\mathrm{d}}{\mathrm{d}t}, \tag{5a}$$

$$n_0=\sqrt{\mu/r_0^3}, \tag{5b}$$

$$\eta=\sqrt{1-e^2}, \tag{5c}$$

$$L=\sqrt{a/r_0}. \tag{5d}$$

Accordingly, the semi-major axis and the eccentricity are constant on average, whereas the inclination, body-fixed longitude of ascending node, argument of periapse, and mean anomaly have secular variations that depend on a, e, i, and Ω_R.

It is worth noting that the latter may become an issue for the design of bounded relative trajectories about strongly elongated bodies. In fact, in order to establish bounded relative motion, one should carefully choose the values of a, e, i, and Ω_R, and try to minimize the difference between the mean orbit element rates of the spacecraft within the formation [8]. However, for any specified value of i and Ω_R, the mean inclination and the mean body-fixed longitude of the ascending node will be changing according to (4c) and (4d) unless i' and Ω_R' are somehow nullified.

As suggested by Hu and Scheeres [6], this is actually possible for orbiters about slowly rotating bodies that satisfy $|w_T/B|<1$, where $B=(3n/2p^2)[2C_{22}-C_{20}]r_0^2$. Then,

$$\Omega_R=\pm\pi/2, \tag{6a}$$

$$\cos i=-\omega_T/B \tag{6b}$$

nullify (4c) and (4d). The relationships (6) are known as the quasi-frozen orbit conditions and can be used in combination with (4) to infer second-order second-degree bounded relative motion conditions.

To that end, consider a formation of two satellites where a, e, i, Ω_R, ω, and M will be referred to as the mean orbit elements of the chief, whereas a_d, e_d, i_d, $\Omega_{R,d}$, ω_d, and M_d will be used to indicate the mean orbit elements of the deputy spacecraft. We will also refer to $\delta i'$, $\delta\Omega_R'$, $\delta\omega'$, and $\delta M'$ as the first variations of $i_d'-i'$, $\Omega_{R,d}'-\Omega_R'$, $\omega_d'-\omega'$, and $M_d'-M'$ respectively.