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Global fuel consumption optimization of an open-time terminal rendezvous and docking with large-eccentricity elliptic-orbit by the method of interval analysis

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ABSTRACT

By defining two open-time impulse points, the optimization of a two-impulse, open-time terminal rendezvous and docking with target spacecraft on large-eccentricity elliptical orbit is proposed in this paper. The purpose of optimization is to minimize the velocity increment for a terminal elliptic-reference-orbit rendezvous and docking. Current methods for solving this type of optimization problem include for example genetic algorithms and gradient based optimization. Unlike these methods, interval methods can guarantee that the globally best solution is found for a given parameterization of the input. The non-linear Tschauner- Hempel(TH) equations of the state transitions for a terminal elliptic target orbit are transformed form time domain to target orbital true anomaly domain. Their homogenous solutions and approximate state transition matrix for the control with a short true anomaly interval can be used to avoid interval integration. The interval branch and bound optimization algorithm is introduced for solving the presented rendezvous and docking optimization problem and optimizing two open-time impulse points and thruster pulse amplitudes, which systematically eliminates parts of the control and open-time input spaces that do not satisfy the path and final time state constraints. Several numerical examples are undertaken to validate the interval optimization algorithm. The results indicate that the sufficiently narrow spaces containing the global optimization solution for the open-time two-impulse terminal rendezvous and docking with target spacecraft on large-eccentricity elliptical orbit can be obtained by the interval algorithm (IA). Combining the gradient-based method, the global optimization solution for the discontinuous nonconvex optimization problem in the specifically remained search space can be found. Interval analysis is shown to be a useful tool and preponderant in the discontinuous nonconvex optimization problem of the terminal rendezvous and docking than GA.

1. Introduction

The impulsive orbit rendezvous strategies between two spacecrafts involve the classical Hohmann rendezvous, double elliptical rendezvous, three-impulse non-coplanar orbital rendezvous and two-impulse Lambert rendezvous [1–4]. Hohmann rendezvous, double elliptical rendezvous, three-impulse non-coplanar orbit rendezvous are all proven techniques based on the impulsive rendezvous of two bodies. The rendezvous between two spacecrafts using these methods only needs several small impulses and the whole impulse Δv is globally optimal while the initial phases of two spacecraft rendezvous need satisfying some specific conditions. The Lambert rendezvous method unlike these methods are suitable for some spacecraft orbit rendezvous missions with unfixed phases, such as space rescue and space interception operations [5,6]. However, these impulsive orbit rendezvous strategies are more suitable for long distance rendezvous than terminal short distance rendezvous. For the terminal short distance impulsive rendezvous and docking, the rendezvous strategies are based on the relative dynamics models of the linear time varying Clohessy-Wiltshire (CW) equations [7,8] and the nonlinear time varying Tschauner- Hempel (TH) equations [9,10].

The relative motions of spacecrafts have been researched since 1950, and the relative motion equations of two spacecrafts were derived by Clohessy and Wiltshire based on the circular two-body reference orbit and named as CW equations [7]. In fact, the approximate relative motion equations with CW equations had been obtained in 1878 by Hill [8]. Thus, the CW equations can be called as Hill equations. The main researches of rendezvous and docking based on the CW equations are the spacecraft terminal approach using the power limited propulsion system. Lembeck and Prussing

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examined the optimum unbounded thrust rendezvous programs for power-limited propulsion systems using the CW equations [11]. Pardis and Carter derived bounded, low-thrust rendezvous trajectories with power-limited propulsion systems based on the CW equations [12] and extended the work for the case where the controller has both upper and lower bounds [13]. Aleshin and Guelman developed a minimum fuel, two-stage solution for the power-limited unbounded thrust rendezvous with a fixed final direction approach via the CW equations [14], and extended the two-stage solution for the case of a power-limited bounded thrust rendezvous with fixed final approach direction [15]. Also, for the elliptical two-body reference orbit, Lawden, Tschauner and Hempel are the first to derive the relative motion equations of spacecraft terminal approach which are named as Lawden equations or TH equations [9,10]. The approximate TH equations are also obtained via incremental changes in orbital elements by Marec [16] and the optimal ellipse-reference-orbit rendezvous [17]. The main researches based on the TH equations are the spacecraft formation flying and the spacecraft terminal approach using the power limited propulsion system. Inalhan and How studied the optimal error box size selection for formation keeping under a known disturbance environment and fuel cost coordination for relative formation changes [18]. Cater examined the optimum unbounded thrust rendezvous programs for power-limited propulsion systems using the TH equations [19]. Therefore, for the terminal short distance impulsive rendezvous and docking, the rendezvous strategies based on whether the CW equations or the TH equations focus on optimizing the fuel cost via the continuous thrust optimal control.

For the terminal rendezvous and docking using the discontinuous and discrete short-time impulsive thrust with finite times, van Kampen studied the fixed-time multiple impulse rendezvous and docking problem based on the CW equations. Thruster pulse amplitudes are optimized by an interval branch and bound algorithm and the cost functions of thruster pulse amplitudes for the terminal rendezvous and docking with the path constraints-obstacles (no-fly region) are also optimized by an interval branch and bound algorithm. These results are compared with those results obtained by the local gradient-based method and the global genetic algorithm (GA) [20]. In fact, the fixed-time terminal impulse rendezvous and docking problem based on the CW equations is a convex optimization problem and can be optimized via local or global optimization algorithms. In this paper, we will extend the fixed-time terminal impulse rendezvous and docking problem based on the circular reference orbit and CW equations to the open-time terminal impulse rendezvous and docking problem based on the elliptical reference orbit and TH equations, that is a discontinuous nonconvex optimization problem. Also, considering the path constraints (sufficient filed of view), unfixed terminal conditions and hybrid cost functions (containing fuel impulse and control period intervals), the discontinuous nonconvex optimization problem will difficult to obtain the global optimization solution. The global GA is an effective method to solve the optimal problem with the convex or nonconvex cost function. However, GA is vulnerable to premature convergence so that the method does not provide the guarantee of global optimal solution for the optimal problem with multiple local minima, and it has been indicated that it can only guarantee to obtain a local minimum for an optimization problem with strong nonlinearity and nonconvexity [21]. Therefore, combining with the local gradient-based optimization method, the deterministicly global optimization algorithm-interval analysis (IA) [22–24] is used to solve the open-time terminal rendezvous and docking optimization problem with strong nonlinearity and nonconvexity. The IA is firstly used to optimize the whole search space to some local sub spaces containing global and some local optimization solutions. The gradient-based optimization method is then used to obtain each optimization solutions via optimizing all remained sub spaces. The global optimization solutions can finally be obtained by comparing all optimization solutions. To avoid interval integral and division, the TH equations in time domain will be transformed into the target spacecraft (TS) true anomaly domain under the TS LVLH frame and the interval states will be constructed involving control amplitudes and open-time.

The organization of this paper proceeds as follows. First, the mathematic modeling for spacecraft terminal relative approach is proposed based on the elliptical reference and TH equations in TS true anomaly domain and TS LVLH frame. Then, the global optimization algorithm for the two-impulse, open-time terminal rendezvous and docking using interval analysis is given. Finally, the numerical simulations are implemented to verify the feasibility of the interval optimization algorithm IA for the discontinuous and nonconvex optimization problem of the two-impulse open-time terminal rendezvous and docking with hybrid cost function, path constraints or unfixed terminal conditions.

2. Mathematic modeling for spacecraft terminal relative approach

For the terminal rendezvous and docking operation phase (not more than a few of tens of kilometers between TS and chase spacecrafts) with the large target orbital arc, the relative orbital dynamics model-TH equations is described in the LVLH frame. Also, the continuous-time relative dynamics equations form time domain to target orbital true anomaly domain is developed to globally optimize the approaching fuel consumption control of CS to TS in this section.

2.1. Coordinate systems

Three coordinate frames, including the geocentric-equatorial inertial frame $\boldsymbol{S}_i$, the geocentric orbital frame $\boldsymbol{S}_o$ and the TS LVLH frame $\boldsymbol{S}_t$, are used for the relative dynamics model.

As seen in Fig. 1, the origin of the equatorial inertial frame $\boldsymbol{S}_i$ locates at the center of the earth, and its three axes $\mathbf{x}_i$, $\mathbf{y}_i$ and $\mathbf{z}_i$ point to the inertial space. The origin of the geocentric orbital frame $\boldsymbol{S}_o$ locates at the center of the earth, the $\mathbf{x}_o$ axis in the TS orbital plane points forward to the TS, the $\mathbf{z}_o$ axis is anticlockwise perpendicular to the TS orbital plane, and the $\mathbf{y}_o$ axis is in the TS

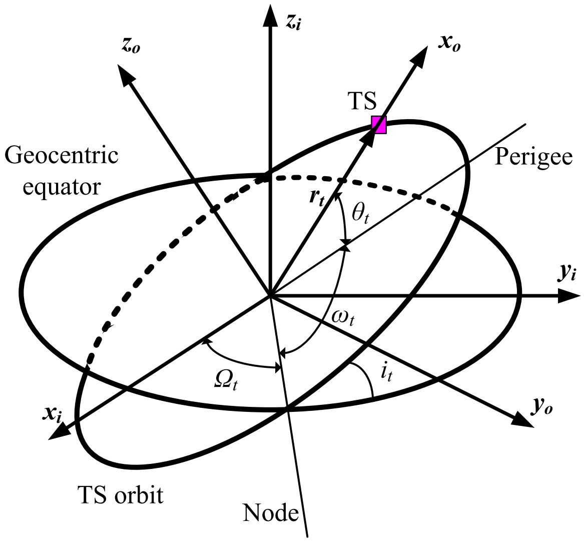

Fig. 1. Coordinate frames for relative dynamics model.