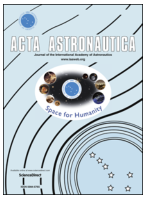
Acta Astronautica 118 (2016) 1–10

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro



Denoising of X-ray pulsar observed profile in the undecimated wavelet domain

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ARTICLE INFO

Article history:
Received 23 December 2014
Received in revised form
18 September 2015
Accepted 30 September 2015
Available online 9 October 2015

Keywords:
X-ray pulsar
Denoising
Signal-dependent noise
Undecimated wavelet transform
LLMMSE

ABSTRACT

The low intensity of the X-ray pulsar signal and the strong X-ray background radiation lead to low signal-to-noise ratio (SNR) of the X-ray pulsar observed profile obtained through epoch folding, especially when the observation time is not long enough. This signifies the necessity of denoising of the observed profile. In this paper, the statistical characteristics of the X-ray pulsar signal are studied, and a signal-dependent noise model is established for the observed profile. Based on this, a profile noise reduction method by performing a local linear minimum mean square error filtering in the un-decimated wavelet domain is developed. The detail wavelet coefficients are rescaled by multiplying their amplitudes by a locally adaptive factor, which is the local variance ratio of the noiseless coefficients to the noisy ones. All the nonstationary statistics needed in the algorithm are calculated from the observed profile, without a priori information. The results of experiments, carried out on simulated data obtained by the ground-based simulation system and real data obtained by Rossi X-Ray Timing Explorer satellite, indicate that the proposed method is excellent in both noise suppression and preservation of peak sharpness, and it also clearly outperforms four widely accepted and used wavelet denoising methods, in terms of SNR, Pearson correlation coefficient and root mean square error.

1. Introduction

Pulsars are highly magnetized, rapidly rotating neutron stars which emit uniquely identifiable signals that are periodical to a high level of accuracy, throughout the electromagnetic spectrum with periods ranging from milliseconds to thousands of seconds. The repetition period of the radiation signals is simply the rotation period of the neutron star. For some pulsars, the stability of their rotation period over long timescales is comparable with terrestrial atomic clocks [1–6]. Of all pulsars, the ones which are visible in the X-ray band of the electromagnetic spectrum are called "X-ray pulsar" [7]. The first X-ray pulsar discovered was Centaurus X-3 in 1967 [8,9]. According to the different energy radiation mechanisms, X-ray pulsars can be divided into rotation-powered, accretion-powered, anomalous X-ray pulsars and so on [3,10,11].

In recent years, X-ray pulsars have drawn much attention of many researchers due to their tremendous research value in astrophysics and broad application prospect in timing and navigation. In astrophysics, X-ray pulsars can be used to probe important relativistic effects such as the frame-dragging and Lense–Thirring effects [12–14] that would play a crucial role in such objects, in a completely different regime with respect to

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http://dx.doi.org/10.1016/j.actaastro.2015.09.018

the few, uncertain and difficult tests performed so far in the weak-field limit of our Solar System [15–17]. For example, X-ray binary pulsars have provided the most accurate examination of Einstein's general theory of relativity so far [18,19]. Besides, X-ray pulsars have unique and periodic pulse profiles, which along with their relatively uniform distribution throughout the sky, suggests their potential use as a natural lighthouse for spacecraft navigation [20–25]. X-ray pulsar-based navigation has been suggested as an attractive approach for autonomous deep space navigation.

The X-ray pulsar's observed profile obtained through epoch folding is of much importance to the research and applications of pulsars [3,5,26–30]. However, since pulsars locate at great distances from earth (e.g., the distance of Crab pulsar from Earth is about 2 kpc [31]), their X-ray pulse radiation, namely the effective radiation, has degenerated to single photon sequence with extremely low intensity when it arrive at the X-ray detector that is mounted on a spacecraft; in addition, the intensity of X-ray background photons, mainly including the diffuse X-ray background [32], the cosmic X-ray background [33,34] and the source shot noise that is contended inherently in the faint source signal [32], is up to about 9 times stronger than that of the source effective photons. These facts lead to low signal to noise ratio (SNR) of the observed profile, and therefore signify the necessity of denoising of the observed profile, especially when the observation time is not long enough [3,30,32].

Wavelet transform has a very useful property of scale and space localization, thus it allows signal feature detection at different scales. In the last decade, wavelets have found successful applications in a variety of signal processing problems, including noise reduction [35]. Some researchers have applied wavelets into denoising of the observed profile of X-ray pulsar, using thresholding techniques which are originally developed on the basis of additive, signal independent Gaussian noise model [36–39]. However, due to the nonstationary characteristic of the detected X-ray pulsar photon signals, the noise of the observed profile is actually signal dependent. This necessarily impacts the denoising performance of the traditional thresholding techniques.

For the signal dependent noise, the minimum mean-square error Wiener filter has been derived and efficiently performed in the wavelet domain by means of a rescaling of the detail coefficients, whose amplitudes are divided by a space-varying factor depending on the SNR of the wavelet coefficients themselves [40].

In this paper, according to the statistical properties of the detected X-ray photon sequence, we introduce a signal-dependent noise model to formulate the observed profile of X-ray pulsar. Based on this model, a local linear minimum mean square error (LLMMSE) filter of the un-decimated wavelet domain is developed to deal with the nonstationary profile noise. Using the parameters of the established signal-dependent noise model and the local statistics estimated from the observed profile, the LLMMSE filter is able to change characteristics according to local signal statistics. The advantage of using undecimated wavelet transform for signal independent noise is that denoising of the undecimated wavelet coefficients is equivalent to a translation-invariant denoising, which can yield better performance than conventional noise reduction methods based on discrete wavelet transform (DWT) [41,42]. The rationale for our choice of denoising in the undecimated wavelet domain is that the classical DWT characterized by downsampling makes the estimation of local statistics of the observed profile critical, owing to the aliasing and lack of translation invariance.

The rest of this paper is organized as follows. In Section 2, a signal-dependent noise model is established for the observed profile. The LLMMSE estimator working in the undecimated wavelet domain is derived in Section 3. In Section 4, experiments are carried out to evaluate this new technique's performance, using both simulated data and real data. Finally, Section 5 concludes the study.

2. Signal dependent noise model of X-ray pulsar observed profile

The original measurement of the X-ray pulsar is the time of arrivals (TOAs) of all the X-ray photons from the pulsar source as well as the background. TOA of a photon is recorded by the X-ray detector when the photon hits the detecting material [43]. To obtain the observed profile, the measured photon TOAs are first transformed to the Solar System Barycenter (SSB) and then assembled into a single pulse cycle through the procedure of epoch folding [3,27,28]. In this paper, to focus our discussion, we assume the photon TOAs have been transformed to SSB. In what follows, according to the presented X-ray detection method and the pulsars' high rotation stability, mathematical equations are used to describe the X-ray pulsar signals; then upon this, statistical properties of the observed profile are given.

2.1. Poisson model of X-ray pulsar signal

Let N_t be the number of arrival photons in the time interval $(0, t)$, $N_0 = 0$ and $t_0 = 0$. Within a certain observation interval, the counting process $\{N(t), t \ge 0\}$ can be approximately modeled by a Cyclostationary Non-Homogeneous Poisson Process (CNHPP) with a periodic time-varying intensity $\lambda(t) \ge 0$ satisfying $\lambda(t) = \lambda(t+T)$, where T is the rotation period of the pulsar. The unit for $\lambda(t)$ is ph/s, which represents number of photons detected per second. $\lambda(t)\ (0 < t < T)$ is also referred to as the standard profile of the pulsar. For a fixed time interval (s, e), the number of arrival photons $N_{s,e}$ is a Poisson random variable with parameter $\int_s^e \lambda(t)dt$. Its distribution law is

$$P(N_{s,e} = k) = \frac{\left(\int_s^e \lambda(t)dt\right)^k \exp\left(-\int_s^e \lambda(t)dt\right)}{k!} \tag{1}$$

and its mean and variance are

$$E[N_{s,e}] = \mathrm{var}[N_{s,e}] = \int_s^e \lambda(t)dt. \tag{2}$$

Furthermore, since $\{N(t), t \ge 0\}$ has independent increments, the numbers of detected photons in any non-overlapping time intervals are independent from each other.

2.2. Epoch folding

The process of epoch folding is to recover the observed profile from the measured photon events [27,28]. It is