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Choosing control parameters for three axis magnetic stabilization in orbital frame

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ABSTRACT

Three-axis magnetic control in orbital reference frame is considered. Magnetorquers are the only actuators and gravitational torque is the only disturbance source. Full attitude knowledge is assumed. Control is constructed on the basis of PD-controller. Stability is analyzed using Floquet theory. Optimal in terms of degree of stability control parameters are found using this approach. These parameters may be further adjusted using numerical simulation. Results for three typical satellites are provided. Numerical example with Kalman filter implementation and additional disturbance sources is provided.

1. Introduction

Three axis magnetic control receives increased attention in present day research. This is due to the rising small satellites industry. Magnetorquers are very attractive actuators for small satellites, especially Cubesats due to low cost, mass, dimensions, power consumption and high reliability. This comes at the cost of underactuation since control torque is perpendicular to the local magnetic induction vector.

Present note supplements our papers [1,2] where relevant survey on the topic can be found. These papers focus on three axis magnetic control in inertial space. It is shown that PD-controller based algorithm provides necessary attitude for restricted control parameters and relevant parameters fitting technique is provided. This technique cannot be used for parameter fitting in orbital stabilization problem due to gyroscopic and gravitational torques. Manual parameter search is time-consuming, especially during mission design process that is prone to frequent satellite properties change.

This note presents semi-analytic method utilizing Floquet theory [3]. Equations of angular motion are linearized in the vicinity of necessary attitude. If they are periodic Floquet theory may be used. That leads to characteristic multipliers of dynamical system. Control parameters are chosen among the best in terms of time-response. Manual adjustment is still desirable to account for complex dynamical model. Recipes for this adjustment are also provided. Corresponding results are present for three typical satellites (Cubesat, micro, few tons monster). Numerical simulation resembling real in-flight performance (Kalman filter, disturbances etc.) is provided.

2. Problem statement

Two right-hand reference frames are used. Orbital frame $OX_1X_2X_3$ constitutes of radius-vector of the satellite OX_1 and orbital normal OX_3. Bound reference frame $Ox_1x_2x_3$ is tied to the principal axes of inertia of the satellite. Mutual attitude is represented with Euler angles ψ, θ, φ (rotation sequence 1-3-2). Direction

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cosines matrix is

$$\mathbf{A}=\begin{pmatrix} \cos\varphi\cos\theta & \cos\varphi\sin\theta\cos\psi+\sin\varphi\sin\psi & \cos\varphi\sin\theta\sin\psi-\sin\varphi\cos\psi \\ -\sin\theta & \cos\theta\cos\psi & \cos\theta\sin\psi \\ \sin\varphi\cos\theta & \sin\varphi\sin\theta\cos\psi-\cos\varphi\sin\psi & \sin\varphi\sin\theta\sin\psi+\cos\varphi\cos\psi \end{pmatrix}.$$

Dynamical Equations of motion utilizing relative velocity $\boldsymbol{\Omega}$ with respect to the orbital frame are

$$\mathbf{J}\frac{d\boldsymbol{\Omega}}{dt}+\boldsymbol{\Omega}\times J\boldsymbol{\Omega}=\mathbf{M}_{rel}+\mathbf{M}_{gr}+\mathbf{M}_{ctrl} \quad (1)$$

where $\mathbf{M}_{rel}=-\mathbf{JWA}\boldsymbol{\omega}_{orb}-\boldsymbol{\Omega}\times\mathbf{JA}\boldsymbol{\omega}_{orb}-\mathbf{A}\boldsymbol{\omega}_{orb}\times\mathbf{J}(\boldsymbol{\Omega}+\mathbf{A}\boldsymbol{\omega}_{orb})$ Here $\mathbf{J}$ is inertia tensor of the satellite; $\boldsymbol{\omega}_{orb}=(0,0,\omega_0)$ is orbital velocity for circular orbit; $\mathbf{M}_{gr}=3\omega_0^2\mathbf{e}_1\times\mathbf{J}\mathbf{e}_1$ is gravitational torque ($\mathbf{e}_1$ is satellite unit radius vector in bound frame); $\mathbf{M}_{ctrl}=\mathbf{m}\times\mathbf{B}$ is control torque ($\mathbf{m}$ is control dipole moment, $\mathbf{B}$ is geomagnetic induction vector in bound frame); $\mathbf{W}$ is skew-symmetric matrix of relative angular velocity

$$\mathbf{W}=\begin{pmatrix} 0 & \Omega_3 & -\Omega_2 \\ -\Omega_3 & 0 & \Omega_1 \\ \Omega_2 & -\Omega_1 & 0 \end{pmatrix}.$$

Kinematics is based on Euler angles

$$\frac{d\psi}{dt}=\frac{1}{\cos\theta}(\Omega_1\cos\varphi+\Omega_3\sin\varphi),$$
$$\frac{d\theta}{dt}=\Omega_3\cos\varphi-\Omega_1\sin\varphi,$$
$$\frac{d\varphi}{dt}=\Omega_2+\mathrm{tg}\theta(\Omega_1\cos\varphi+\Omega_3\sin\varphi). \quad (2)$$

Direct and inclined dipole models are used for geomagnetic field representation. Direct dipole model leads to periodic right side of equations of motion which is crucial for Floquet theory implementation. Geomagnetic induction vector in orbital reference frame is

$$\mathbf{B}=\frac{\mu_e}{r^3}\begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix}=\frac{\mu_e}{r^3}\begin{pmatrix} -2\sin u\sin i \\ \cos u\sin i \\ \cos i \end{pmatrix}$$

where $\mu_e=7.812\cdot10^6\ \mathrm{km^3\ kg\ s^{-2}\ A^{-1}}$, r is satellite radius vector magnitude. Inclined dipole model is used in numerical simulation to represent geomagnetic field with greater accuracy. This also allows us to test approximate results applicability to a more "real" mathematical model of satellite motion.

Control dipole moment is common PD-controller based algorithm

$$\mathbf{m}=-k_\omega\mathbf{B}\times\boldsymbol{\Omega}-k_a\mathbf{B}\times\mathbf{S} \quad (3)$$

where $\mathbf{S}=(a_{23}-a_{32},\ a_{31}-a_{13},\ a_{12}-a_{21})$, k_ω and k_a are positive damping and positional control gains. Two approaches for this control construction in inertial frame are present in [1]. Both do not take into account gravitational and gyroscopic torques. The last one is not zero in necessary position where it equals $\mathbf{A}\boldsymbol{\omega}_{orb}\times\mathbf{JA}\boldsymbol{\omega}_{orb}$. Three axis magnetic control (3) is nevertheless quite efficient for orbital attitude as is shown using Floquet theory.

3. Floquet analysis

Floquet theory is used to assess satellite behavior with wide range of control parameters k_a, k_ω and its inertia moments (hence gravitational and gyroscopic torques). Equations of motion (1) and (2) are linearized in the vicinity of necessary attitude that corresponds to bound and orbital frames coincidence,

$$\frac{d\omega_1}{du}=-K_\omega\frac{B_0^2}{A\omega_0^2}\left[\left(B_2^2+B_3^2\right)\omega_1-B_1B_2\omega_2-B_1B_3\omega_3\right]$$
$$-2k_a\frac{B_0^2}{A\omega_0^2}\left[-B_1B_2\varphi-B_1B_3\theta+\left(B_2^2+B_3^2\right)\psi\right]+\omega_2+\frac{B-C}{A}(\omega_2+\psi),$$

$$\frac{d\omega_2}{du}=-K_\omega\frac{B_0^2}{B\omega_0^2}\left[-B_1B_2\omega_1+\left(B_1^2+B_3^2\right)\omega_2-B_2B_3\omega_3\right]$$
$$-2k_a\frac{B_0^2}{B\omega_0^2}\left[\left(B_1^2+B_3^2\right)\varphi-B_2B_3\theta-B_1B_2\psi\right]-\omega_1+\frac{C-A}{B}(\omega_1-4\varphi),$$

$$\frac{d\omega_3}{du}=-K_\omega\frac{B_0^2}{C\omega_0^2}\left[-B_1B_3\omega_1-B_2B_3\omega_2+\left(B_1^2+B_2^2\right)\omega_3\right]$$
$$-2k_a\frac{B_0^2}{C\omega_0^2}\left[-B_2B_3\varphi+\left(B_1^2+B_2^2\right)\theta-B_1B_3\psi\right]+3\frac{A-B}{C}\theta,$$

$$\frac{d\varphi}{du}=\omega_2,\quad \frac{d\theta}{du}=\omega_3,\quad \frac{d\psi}{du}=\omega_1. \quad (4)$$

Damping parameter k_ω is substituted with $K_\omega=k_\omega\omega_0$. This makes both control gains comparable and justifies dimensionless relative angular velocity components $\omega_i=\Omega_i/\omega_0$ in linearized Eq. (4). Derivatives are taken with respect to the argument of latitude $u=\omega_0t+u_0$. Homogeneous part in Eq. (4) is the same as for inertial attitude [1] but

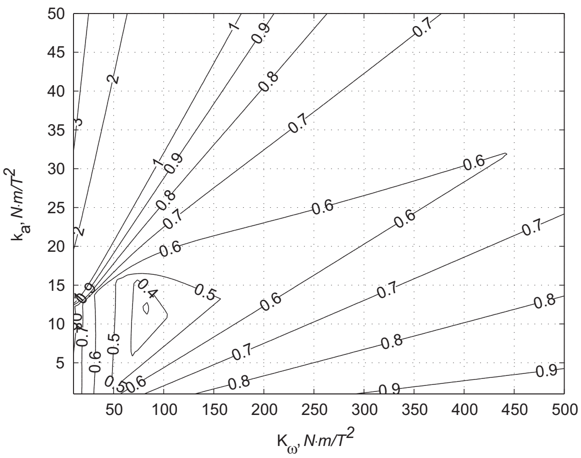

Fig. 1. Characteristic exponents, Cubesat.