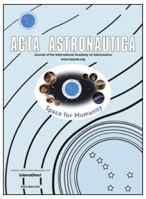
Acta Astronautica 116 (2015) 117–125

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro



# Large amplitude nonlinear vibration analysis of functionally graded Timoshenko beams with porosities

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## ARTICLE INFO

*Article history:*
Received 8 March 2015
Received in revised form
5 June 2015
Accepted 16 June 2015
Available online 25 June 2015

*Keywords:*
Large-amplitude
Nonlinear free vibration
Porous material
Functionally graded material
Timoshenko beam

## ABSTRACT

In this paper, the large-amplitude nonlinear vibration characteristics of functionally graded (FG) Timoshenko beams made of porous material is investigated for the first time. Material properties of FG porous beam are supposed to vary continuously along the thickness according to the rule of mixture which modified to approximate material properties with porosity phases. The governing equations are derived based on Timoshenko beam theory through Hamilton's principle and they are solved utilizing both Galerkin's method and the method of multiple scales. According to the numerical results, it is revealed that the proposed modeling can provide accurate frequency results of the FG porous beams as compared to the literature. The detailed mathematical derivations are presented and numerical investigations are performed while the emphasis is placed on investigating the effect of the several parameters such as material distribution profile, porosity volume fraction, aspect ratio and mode number on the normalized natural frequencies of the FG porous beams in detail. It is explicitly shown that the vibration behavior of a FG beams is significantly influenced by these effects. Numerical results are presented to serve as benchmarks for future analyses of FG porous beams.

## 1. Introduction

Space planes require high-performance heat-resistant materials which can withstand ultrahigh temperatures and extremely large temperature gradients. To meet these needs, functionally gradient materials (FGMs) were proposed in the early 1980s in Japan [1]. When the space shuttles are accelerating the outer space atmosphere, the surface temperature of the plane could be as high as 2100 K due to the acute friction. It requires the material with excellent- heat resistance features to withstand high temperature and thermal impact in one side while on the other side (liquid hydrogen-cooled), it requires materials with good thermal conductivity and toughness to ensure rapid cooling and a certain lifetime. The advent of Ceramic–metal materials made an excellent solution to this problem. Ceramics settled in the outer side can withstand high temperature while in other side metal is used [2]. With the continuous variation of the material properties along the thickness, FGMs can be used in high temperature gradient environments especially when they are made of metal and ceramic. The metal can keep a certain extent of toughness and ceramics have superior heat resistant ability. So they usually act as thermal protection structures in spacecraft and other structural components in high temperature environments. With these superior characteristics, FGMs are proposed as one of the main materials families for thermal protection applications that are being developed for the fourth generation of future NASA Reusable Launch Vehicles within NASA [3]. FGMs, are achieved by controlling the volume fractions, microstructure, porosity, etc. of the material constituents during manufacturing, resulting in spatial gradient of macroscopic material properties of

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http://dx.doi.org/10.1016/j.actaastro.2015.06.014
0094-5765/

mechanical strength and thermal conductivity [4]. The material properties of FGMs change smoothly between two surfaces and the advantages of this combination lead to novel structures which can withstand in large mechanical loadings under high temperature environments [5]. Presenting novel properties, FGMs have also attracted intensive research interests, which were mainly focused on their static, dynamic and vibration characteristics of FG structures [6,7]. Therefore, FGMs have received wide applications in modern industries including aerospace, mechanical, electronics, optics, chemical, biomedical, nuclear, and civil engineering to name a few during in the past two decades [8,9].

Based on the open literature, it is found that many researchers have investigated the vibration response of FGM beams in the framework of linear analysis. For example, Aydogdu and Taskin [10] presented linear frequencies of simply supported FGM beams. Sina et al. [11] employed modified and traditional first order shear deformation theories to deal with vibration problem of FGM beams, using an analytical method. Şimşek [12] used several beam theories to analyze vibration response of FGM beams supported by different general boundary conditions. An improved third order shear deformation theory was employed by Wattanasakulpong et al. [13] to investigate thermal buckling and thermo-elastic vibration of FGM beams with different immovable boundary conditions. The buckling and vibration responses of FG nanoplates that subjected to thermal loading was studied by Ansari et al. [14]. Ebrahimi and Salari [15] analyzed free vibration of FG nanobeams in thermal environment. In all of these linear studies the effect of porosities in FGM was ignored.

Besides due to porosities occurring inside FGMs during fabrication, it is therefore necessary to consider the vibration behavior of beams having porosities in this investigation. The porous materials are composed of two elements: one of which is solid (body) and the other element is either liquid or gas that is frequently found in nature, such as wood, stone, and layers of dust. For many years, porous material structures, such as beams, plates, and shells, have been widely discussed in structural design problems. Ebrahimi and Mokhtari [16]. Wattanasakulpong et al. [17] analyzed free vibration of layered functionally graded beams and validated his results with experimental results; he concluded that discrepancies between theoretical and experimental results could arise from porosities due to imperfect infiltration and from approximation in material profile in calculation. Recently Ebrahimi and Mokhtari [16] studied the transverse vibration behavior of rotating porous FGM beam. Both of these studies are based on linear vibration analysis only. Most recently Wattanasakulpong and Ungbhakorn [18] studied Linear and nonlinear vibration analysis of elastically restrained ends FGM beams. They used a modified form of rule of mixture to describe material properties of FG beam with porosities. But they utilized Euler–Bernoulli beam theory (EBT) which ignores the effects of shear deformation and rotary inertia. Due to this cause, the EBT always overestimates the natural frequency of free vibration. Moreover, Timoshenko beam theory presents a more realistic model of the beam in the determination of higher modes of vibration. Long slender beams could be modeled with EBT since the flexural behavior is dominant while application of EBT leads to great inaccuracy in modeling short thick beams where shear deformation is more significant [19].

As a result, there are few literatures devoted to the nonlinear free vibration of non-homogeneous FG Timoshenko beams especially with porosities. To the best knowledge of the authors, no research effort has been devoted so far to find the solution of nonlinear vibrational behavior of a porous FG Timoshenko beam. Motivated by these considerations, in the present study, nonlinear free vibration of a porous beam composed of functionally graded materials is investigated in conjunction with Timoshenko beam theory. The governing differential equations of motion are derived using the Hamilton's principle and the Galerkin's method along with the method of multiple scales is utilized to obtain a solution to the nonlinear transverse vibration problem. Some illustrative numerical examples are presented in order to investigate the influences of FG material volume fraction index and porosity parameter on natural frequencies of tapered rotating FG beam in detail. To verify the present analysis, the results of this study are compared with the available results from the existing literature and an excellent agreement is observed.

## 2. Functionally graded beams with porosities

A clamped–clamped FG beam of length $(l)$, width $b$, thickness $(h)$, is considered in this investigation. In this study, it is assumed that the FG beam is made of ceramic and metal, and the effective material properties of the FG beam, i.e., Young's modulus $E$, Poisson's ratio $\nu$, shear modulus $G$ and material density $(\rho)$, vary continuously in the thickness direction according to a function of the volume fractions of the constituents. In this investigation, the imperfect beam is assumed to have porosities spreading within the beam cross-section due to defect during production. Consider an imperfect FGM with a porosity volume fraction, $\alpha$ $(\alpha < < 1)$, distributed evenly among the metal and ceramic, our modified rule of mixture is proposed as

$$P = P_m\left(V_m - \frac{\alpha}{2}\right) + P_c\left(V_c - \frac{\alpha}{2}\right) \tag{1a}$$

Now, the total volume fraction of the metal and ceramic is: $V_m + V_c = 1$, and the power law of volume fraction of the ceramic is described as

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{1b}$$

Hence, all properties of the imperfect FGM can be written as

$$P = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + P_m - (P_m - P_c)\frac{\alpha}{2} \tag{2}$$

It is noted that the positive real number $n$ $(0 \le n < \infty)$ is the power law or volume fraction index, and $z$ is the