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# How can we increase the accuracy of determination of spacecraft's lifetime?

A.I. Nazarenko

*Scientific Technological Center "COSMONIT", Moscow, Russia*

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ABSTRACT

In their content, the materials of this article represent continuation of some earlier published results of studies aimed at increasing the accuracy of determination and forecasting of the orbits of low satellites [1–7]. All these materials consider the application of the modification of the maximum likelihood technique called the technique of optimum filtering the measurements (OFM).

A feature of presented materials is the determination and forecasting of parameters of the "Tchibis-M" spacecraft orbit before the reentry instant over a rather long time interval (about 10 months). This makes it possible to more fully estimate the effect of atmospheric disturbances on the calculation results and to develop recommendations on increasing the accuracy of solution of the problem in question.



## 1. Introduction

The technique for solution of the problem under consideration is based on integration of the equations of motion with the known initial data. It consists of six dimensional status vectors and estimation of the drag parameter. Various characteristics are used as a drag parameter. Estimates of ballistic coefficient (*Sb*) and variations in the revolution period under atmospheric effects ($\Delta T$) are most popular.

To determine the initial data from measurements, the least square technique (LST) is commonly used. This technique was developed 200 years ago, when artificial satellites did not exist yet. The motion of an artificial satellite is highly affected by disturbing factors, which cannot be described mathematically to a necessary accuracy. The atmospheric drag is a typical example of such a disturbance; its value is proportional to the product of a real ballistic coefficient and atmospheric density. These factors change in time unpredictably; that is why it is very difficult to take them into account during prediction. When using the LST, the effect of disturbing factors is revealed in the choice of the optimum (measuring) range, i.e., the time interval, during which the measurements are carried out. The studies have shown that the optimum value depends not only on the drag parameter, but also on the accuracy and number of measurements. In practice, this range is usually found experimentally and is fixed for specific types of satellites.

The author published the principles of the applied procedure almost 40 years ago [7]. In 1970s this procedure was implemented at the Russian Space Surveillance Center for determining and forecasting the orbits of low satellites. Subsequently this procedure was updated. The characteristic feature of the procedure is accounting for statistical characteristics of atmospheric disturbances over the fit span and during the motion forecasting. The results of studying atmospheric disturbances were published in a number of articles, for example, in [8].

*E-mail address:* anazarenko32@mail.ru

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Modeling of the algorithm and software for the optimum measurement filtering demonstrated the efficiency of the method. Though the OMF basis was developed rather long ago, its wide practical application was restrained by a series of circumstances. One of the circumstances was insufficient knowledge of atmospheric density variations and ballistic characteristics of space objects. Another circumstance was associated with rather poor characteristics of computer technology (memory, speed, word length).

## 2. Optimum filtering the measurements

In the OFM technique the problem of evaluating the state vector $x$ (n × 1) based on the measurements $Z$ (k × 1) is considered in the classical formulation. The possibility of existence of some interfering parameters $q$ (m × 1) is taken into consideration. In this case the basic initial relationship for measurements $Z$ takes the form

$$Z = X \cdot x + B \cdot q + V \tag{1}$$

Here $X$ (k × n) and $B$ (k × m) are the known matrices, $V$ (k × 1) is the vector of errors in measurements which are assumed to be equally accurate and statistically independent variables, i.e.

$$M\left(V \cdot V^T\right) = \sigma_z^2 \cdot E \tag{2}$$

where $E$ is a unit matrix (k × k).

The correlation matrix of interfering parameters $M(q \cdot q^T) = \sigma_q^2 \cdot K_{q\Sigma}$ (m × m) is assumed to be known. It is constructed taking into account the correlation of atmospheric disturbances and is used for "weighting" the measurements without expanding the state vector. The effect of interfering parameters is taken into consideration by their combining with the errors of measurements ($V_\Sigma = B \cdot q + V$), and then the maximum likelihood technique is applied. In this case the required estimate is expressed as follows:

$$\widehat{x} = \left(X^T \cdot P \cdot X\right)^{-1} \cdot X^T \cdot P \cdot Z, \tag{3}$$

where the weighting matrix is

$$P = \left(\frac{\sigma_q^2}{\sigma_z^2} \cdot B \cdot K_{q\Sigma} \cdot B^T + E\right)^{-1} = \left(S_n^2 \cdot B \cdot K_{q\Sigma} \cdot B^T + E\right)^{-1} \tag{4}$$

Here parameter $S_n = \sigma_q / \sigma_z$ can be treated as the signal-to-noise ratio. The estimate (3) provides the maximum of the likelihood function and the minimum of the criterion

$$Criterion = \sqrt{(Z - X \cdot \widehat{x})^T \cdot P \cdot (Z - X \cdot \widehat{x})/k}. \tag{5}$$

The feature of estimate (3) is the fact that it is suitable for any time instants, including forecasting one. The value of interfering parameters (noises) is calculated after constructing the estimate (3) on the basis of residual discrepancies with the using the relationship of the form

$$\hat{q} = F \cdot (Z - X \cdot \widehat{x}) \tag{6}$$

where $F$ is some matrix.

Another feature of the OFM technique is the necessity of inversion of matrix (4) of dimension (k × k). With contemporary characteristics of computer technology this operation is quite feasible.

The autocorrelation function of atmospheric disturbances is assumed to be of the form

$$K_q(t,\tau)_0 = \begin{cases} \sigma_q^2\left(1 - \frac{|t-\tau|}{\Delta}\right), & by \quad |t-\tau| < \Delta, \\ 0 \quad by & |t-\tau| \geq \Delta. \end{cases} \tag{7}$$

The initial data for applying this correlation function are

$\Delta T$ – the change of the period under an effect of atmospheric drag per revolution, which is calculated on the basis of numerical integration with the mean value of ballistic coefficient;
$k_{atm}$ – the RMS of random atmospheric disturbances with respect to their mean value;
$\Delta$ – the interval of correlation of atmospheric disturbances.

The first two quantities are used for calculating the RMS of atmospheric drag variations according to the formula

$$\sigma_q = k_{atm} \cdot |\Delta T| \tag{8}$$

The matrices of cross-correlation of errors in forecasting the state vector at time instants ($t_i$ and $t_l$) are calculated according to the formula

$$M\left[\delta x(t_i) \cdot \delta x^T(t_l) | x(t_k)\right] = \int_{t_k}^{t_i} \int_{t_k}^{t_l} U(t_i,\xi) \cdot B(\xi) \cdot K_q(\xi,\eta)_0 \cdot B^T(\eta) \cdot U^T(t_l,\eta) \cdot d\eta \cdot d\xi \tag{9}$$

Here $U(t_i, \xi)$ is the so-called transition matrix of dimension (6 × 6), $B(\xi)$ is the matrix of coefficients at atmospheric drag in the differential equations of the disturbed motion.

The consideration of atmospheric disturbances in updating the orbits is manifested in a substantially different (as compared to the least square technique (LST)) behavior of residual discrepancies between the measured and updated orbital parameters over the fit span (Table 1). The example relates to processing the measurements for the launch vehicle (the rocket) that was separated from the "Phobos-Grunt" SC at launching [5].

It is seen from the table that, with using the OFM technique, the residual discrepancies very highly change on the fit span. The main effect of applying the OFM technique consists in the increase of the accuracy of orbit determination at the last point of a fit span, i.e. at the instant of receiving the initial data (ID) for forecasting. In this case the decrease of the level of residual discrepancies is almost

**Table 1**
RMS of residual discrepancies in time (s) with using the LST and the new technique (optimum filtering of measurements).

| Technique | Numbers of measurements over the fit span | | | | | | |
|---|---|---|---|---|---|---|---|
| | k-6 | k-5 | k-4 | k-3 | k-2 | k-1 | k |
| LST | – | – | 0.315 | 0.712 | 0.669 | 0.789 | **0.394** |
| OFM | 18.749 | 14.785 | 11.460 | 7.799 | 5.534 | 1.751 | **0.081** |