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Unified kinematic framework for a non-nominal Euler axis/angle rotation

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ABSTRACT

In this paper, kinematics equations of attitude parameters are derived for cases where the Euler rotation theorem cannot be applied and the single rotation that takes an initial reference frame to a target reference frame cannot be performed. It is the case when the nominal rotation is not allowed along a prescribed direction in space, namely an "underactuation" direction. As a matter of fact, a non-nominal maneuver planning scheme, expressed in terms of Euler axis/angle parameters, is admitted for the minimization of the alignment error between the target and the attainable attitude. The derivation of kinematic equations, describing the time evolution of non-nominal rotation parameters, is performed by means of rigorous algebraic manipulations within a unified framework, where the underactuation direction is prescribed in either the moving frame or the target frame.

1. Introduction

Euler axis/angle representation is a widely used technique in rigid body attitude control problems. It allows the visualization of the nominal rotation which takes a rotating reference frame (e.g., a body-fixed frame) to a target reference frame by means of the minimum angular path [1]. There are cases where there exists a direction, along the unit vector $\hat{\boldsymbol{b}}$, about which rotations are not allowed, making the desired Euler transformation about the nominal axis $\hat{\boldsymbol{e}}$ not attainable. Nonetheless, under such underactuated conditions, rotations about non-nominal axes, lying on the plane orthogonal to $\hat{\boldsymbol{b}}$, can be performed.

In [2] one of the authors provided an exact analytical expression allowing to compute the rotation angle, $\hat{\phi}$, about the instantaneous non-nominal rotation axis, $\hat{\boldsymbol{g}} = \left(\hat{\boldsymbol{b}} \times \hat{\boldsymbol{e}} / \left\|\hat{\boldsymbol{b}} \times \hat{\boldsymbol{e}}\right\|\right) \times \hat{\boldsymbol{b}}$, which minimizes the alignment error between the target and the attainable attitude. More recently, Avanzini and Giulietti [3] also demonstrated that it is always possible to determine a non-nominal Euler axis/angle rotation driving a single axis of the rotating frame to be aligned with a prescribed direction in space.

The present study, based on the conference paper in [4], is aimed to complete the kinematic framework of Euler axis/angle representation in the presence of constraints on the direction of admissible rotation axes. Kinematic equations, describing the time evolution of non-nominal rotation parameters in the case when the overall misalignment error is minimized, are derived. A novel approach is presented allowing a generalized solution to the problem of underactuation when the torqueless direction is constant in either the target or the body frame. Examples of suitable applications of the proposed approach may be found in microsatellite platforms. In case of magnetic attitude control, the body-referenced available control torque is always perpendicular to the external magnetic field (whose direction is supposed to be fixed in space in

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short timescales) and to the onboard-generated magnetic dipole. This makes the system inherently underactuated, with the inability to provide three independent control torques at each time instant [5]. On the other hand, the spacecraft can become underactuated after a failure in a minimal control system or multiple failures in a redundant one (i.e., after the loss of a reaction wheel in a three-axes stabilized spacecraft with no redundancy [6]). In these cases, the application of well known control strategies is no longer possible for both regulation and tracking, and new methods have been proposed for tackling this particular problem [7]. In this respect, the present work addresses a unified framework of mathematical tools suitable for dealing with underactuation directions both fixed in space and in the body-fixed frame. Three-axis control system design where the non-nominal Euler axis $\hat{\boldsymbol{g}}$ and the relative rotation angle $\hat{\phi}$ are used as feedback terms represent a suitable application.

In what follows a brief overview about nominal Euler axis/angle representation is provided in the first part of Problem Formulation Section, while the non-nominal rotation planning scheme is introduced in the second one. The derivation of kinematic equations for the non-nominal rotation is then analyzed for the two cases in which the torqueless direction $\hat{\boldsymbol{b}}$ is a constant in the rotating and the target frame, respectively. A section of concluding remarks ends the paper.

2. Problem formulation

2.1. Nominal Euler axis/angle rotation

Define two arbitrary Cartesian coordinate frames: a rotating reference frame, $\mathbb{F}_1$, and a target reference frame, $\mathbb{F}_2$. Let $\mathbb{T}_{12} \in \mathbb{R}^{3\times 3}$ represent the rotation matrix that allows for the transformation

$$\boldsymbol{v}_1 = \mathbb{T}_{12}\boldsymbol{v}_2 \tag{1}$$

where $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ represent a generic vector expressed in $\mathbb{F}_1$ and $\mathbb{F}_2$, respectively. Suppose that the desired attitude is achieved when $\mathbb{F}_1$ is aligned with the target frame $\mathbb{F}_2$. According to Euler's Theorem, this can be obtained by a single rotation of the frame $\mathbb{F}_1$ about an axis referred to as the Euler axis (or rotation eigenaxis), whose components do not depend on the particular reference frame ($\mathbb{F}_1$ or $\mathbb{F}_2$). In what follows, all vector components will be expressed in $\mathbb{F}_1$, unless noted otherwise.

Let $\hat{\boldsymbol{e}} \in \mathbb{R}^3$ represent the Euler axis unit vector and let $\phi \in (0, \pi)$ represent the Euler angle of rotation about the Euler axis. Euler axis and angle can be expressed as a function of the rotation matrix as follows [8]:

$$\cos\,\phi = \tfrac{1}{2}[\mathrm{tr}(\mathbb{T}_{12}) - 1] \tag{2}$$

$$\hat{\boldsymbol{e}}^{\times} = \frac{1}{2\,\sin\,\phi}(\mathbb{T}_{12}^{T} - \mathbb{T}_{12}) \tag{3}$$

where $\mathrm{tr}(\mathbb{T}_{12})$ is the trace of $\mathbb{T}_{12}$ and $\hat{\boldsymbol{e}}^{\times}$ is the skew symmetric cross-product operator

$$\hat{\boldsymbol{e}}^{\times} = \begin{bmatrix} 0 & -e_3 & e_2 \\ e_3 & 0 & -e_1 \\ -e_2 & e_1 & 0 \end{bmatrix} \tag{4}$$

related to the vector components of $\hat{\boldsymbol{e}}$. On the converse, the reciprocal equations lead to an expression of the rotation matrix in terms of Euler axis/angle:

$$\mathbb{T}_{12} = \boldsymbol{I}_3 - \sin\,\phi\,\hat{\boldsymbol{e}}^{\times} + (1 - \cos\,\phi)\hat{\boldsymbol{e}}^{\times}\hat{\boldsymbol{e}}^{\times} \tag{5}$$

provided $\boldsymbol{I}_3$ is the 3×3 unit matrix.

The kinematics of Euler axis/angle representation when the angular velocity is known is provided. Let $\boldsymbol{\omega}_{21} \in \mathbb{R}^3$ be the angular velocity of $\mathbb{F}_1$ relative to $\mathbb{F}_2$, expressed in $\mathbb{F}_1$. It is [9]

$$\dot{\phi} = \hat{\boldsymbol{e}} \cdot \boldsymbol{\omega}_{21} \tag{6}$$

where $\hat{\boldsymbol{e}} \cdot \boldsymbol{\omega}_{21}$ is the scalar product between $\hat{\boldsymbol{e}}$ and $\boldsymbol{\omega}_{21}$, and

$$\dot{\hat{\boldsymbol{e}}} = \frac{1}{2}\left[\hat{\boldsymbol{e}}^{\times} - \cot\left(\frac{\phi}{2}\right)\hat{\boldsymbol{e}}^{\times}\hat{\boldsymbol{e}}^{\times}\right]\boldsymbol{\omega}_{21} \tag{7}$$

2.2. Non-nominal Euler axis/angle rotation

Given a generic attitude achievable by a rotation $\phi \neq 0, \pi$ about the eigenaxis $\hat{\boldsymbol{e}}$ (the two cases $\phi = 0, \pi$ represent singularities in the Euler's Theorem, see Eq. (3), and therefore must be excluded), there could be cases where the desired rotation $\mathcal{R}_{des}(\hat{\boldsymbol{e}}, \phi)$ cannot be performed. Let $\hat{\boldsymbol{g}} \in \mathbb{R}^3$ be a unit vector not aligned to the Euler axis $\hat{\boldsymbol{e}}$: a rotation about $\hat{\boldsymbol{g}}$ by the angle ϕ would take $\mathbb{F}_1$ to a frame $\mathbb{F}_3$ which is not aligned to the target frame $\mathbb{F}_2$.

In [2] an analytical expression was derived for the particular rotation angle $\hat{\phi} \in [0, \pi)$, about a generic axis $\hat{\boldsymbol{g}}$ not aligned to $\hat{\boldsymbol{e}}$, which minimizes the alignment error ϵ between the target frame $\mathbb{F}_2$ and the attainable frame $\mathbb{F}_3$ (see Fig. 1). In particular, it was proven that [2,10]

$$\tan\left(\frac{\hat{\phi}}{2}\right) = (\hat{\boldsymbol{e}} \cdot \hat{\boldsymbol{g}})\,\tan\left(\frac{\phi}{2}\right) \tag{8}$$

If, in addition, the rotations are constrained on a plane orthogonal to a unit vector $\hat{\boldsymbol{b}} \in \mathbb{R}^3$, it was shown how the best admissible rotation $\mathcal{R}(\hat{\boldsymbol{g}}, \hat{\phi})$ leading to the minimum misalignment error, $\epsilon = \epsilon(\hat{\boldsymbol{b}}, \mathbb{T}_{12})$, is obtained when the

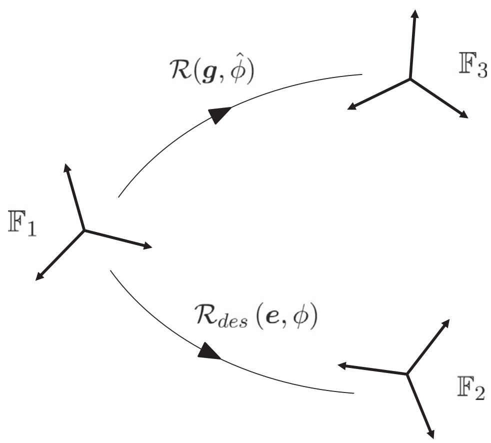

Fig. 1. Definition of desired and admissible rotations.