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Analytical study of microsatellite attitude determination algorithms

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ABSTRACT

An analytical approach to study of attitude determination algorithms is considered. The approach is applicable for quasi-stationary motion determination. It is based on filter post-convergence computation of the Kalman filter covariance matrix and allows one to estimate the influence of unaccounted perturbations on motion determination accuracy. The dependence of attitude determination accuracy on filter parameters and perturbations is obtained. The proposed method of improving the Kalman filter performance is applied on board a microsatellite of the TabletSat series.

1. Introduction

Microsatellites are now widely used for Earth observation [1,2] and scientific missions [3]. They are also considered promising for interplanetary flights [4,5]. Use of attitude control systems is crucial for most of the microsatellite missions. One of the main issues is a problem of estimating the attitude motion state vector. It is often required in real time. The state vector is determined by processing attitude sensors measurements by the onboard computer which has computational constraints due to the power limitation. This problem is commonly solved using recursive algorithms based on the Kalman filter.

The most frequently used microsatellite attitude determination sensors are star sensors [6,7], sun sensors [8], gyros [9,10] and magnetometers [11,12]. Each type of sensors has its strengths and shortcomings which determine its choice in specific attitude motion modes. For example, star sensors are considered to be the most accurate but they are to be used only at small angular velocities and when the Sun is out of the field of view. Sun sensors are completely useless on the shaded part of the orbit. A magnetometer can only be exploited in low-Earth orbits and its measurements are disturbed by the proper magnetic field of a microsatellite. An angular velocity sensor requires real-time calibration because of the changing bias. Thus, it is reasonable to use all types of sensors and to process the most reliable (under current conditions) measurements. The most common set of microsatellite sensors includes a sun sensor, a magnetometer and an angular velocity sensor. The sun sensor and magnetometer measurements should be processed on the sunlit part of the orbit while the angular velocity sensor undergoes calibration. Then the magnetometer and the calibrated angular velocity sensor are used on the shaded part [3]. Microsatellite attitude determination and control system (ADCS) should therefore contain a set of different attitude determination algorithms for estimating the attitude state vector.

The Kalman filter uses the linearized satellite attitude motion equations and sensor measurements to estimate the state vector by the mean-square criterion. Due to computational constraints, the motion equations cannot include all perturbations acting on a satellite. The attitude

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determination accuracy decreases as unaccounted perturbations influence grows. Selection of motion model error and measurement noise statistics, commonly known as filter tuning, is a critical issue for the Kalman filter. In the recent paper [13], we presented an overview of Kalman filter tuning methods and proposed an analytical approach to study the filter performance. This approach is applicable for quasi-stationary motion analysis. In the present paper, we develop the latter work [13] and apply the approach to study the algorithms based on the readings of a star sensor, a sun sensor, a magnetometer and an angular velocity sensor. The primary purposes of the paper are the analytical study and comparison of the attainable accuracy of attitude determination algorithms based on the measurements of various sensors sets. The parameters of ADCS sensors of TabletSat microsatellite series[1] are considered as an example.

2. Kalman filter tuning technique

2.1. Extended Kalman filter

First, consider briefly the well-known extended Kalman filter algorithm [14,15]. Assume the satellite motion model to be nonlinear

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t) + \mathbf{q} \quad (1)$$

where $\mathbf{x}$ is a state vector, $\mathbf{f}$ is a nonlinear function, $\mathbf{q}$ is a normally distributed dynamical noise with the error covariance matrix Q. Prediction of the state vector estimation $\hat{\mathbf{x}}_{k+1}^-$ (*a priori*) at the moment of time t_{k+1} is calculated by integration of nonlinear Eq. (1) (without $\mathbf{q}$ vector) using the state vector $\hat{\mathbf{x}}_k^+$ at previous time step t_k. Discrete Riccati equation is used to obtain prediction of the error covariance matrix vector estimation P_{k+1}^- at time t_{k+1},

$$P_{k+1}^- = \Phi_k P_k^+ \Phi_k^T + Q, \quad (2)$$

where Φ_k is a transition matrix between the states $\mathbf{x}_k$ and $\mathbf{x}_{k+1}$ which is calculated by linearizing Eq. (1) in the neighborhood of $\hat{\mathbf{x}}_k^-$, P_k^+ is an error covariance matrix at t_k.

A posteriori estimation is *a priori* estimation corrected by the measurements sample. In general case, the measurement vector $\mathbf{z}$ depends nonlinearly on the state vector $\mathbf{x}$,

$$\mathbf{z}_{k+1} = \mathbf{h}(\mathbf{x}_{k+1}^-, t_{k+1}) + \mathbf{r}. \quad (3)$$

Here $\mathbf{h}$ is a nonlinear function, $\mathbf{r}$ is a measurement noise vector with the covariance matrix R. The gain matrix K_k can be written as

$$K_{k+1} = P_{k+1}^- H_{k+1}^T [H_{k+1} P_{k+1}^- H_{k+1}^T + R]^{-1} \quad (4)$$

where H_{k+1} is a sensitivity matrix calculated by linearizingmeasurement model (3) in the neighborhood of $\hat{\mathbf{x}}_k^-$. The corrected (*a posteriori*) estimation $\mathbf{x}_{k+1}^+$ of the Kalman filter has the form

$$\mathbf{x}_{k+1}^+ = \mathbf{x}_{k+1}^- + K_{k+1}[\mathbf{z}_{k+1} - \mathbf{h}(\mathbf{x}_{k+1}^-, t_k)].$$

A posteriori estimation for the error matrix is given by the formula

$$P_{k+1}^+ = [E - K_{k+1} H_{k+1}] P_{k+1}^-$$

where E is an identity matrix.

2.2. Accuracy estimation in quasi-stationary motion

The Kalman filter covariance matrix of errors P is a qualitative criterion of the state vector estimation. If the matrix P_k at time t_k is known, one can estimate the accuracy of determining the state vector $\hat{\mathbf{x}}_k$. However, the value of P_k depends on a number of factors like the initial state vector $\mathbf{x}_0$, initial value P_0, covariance matrix of motion model error Q, measurement errors R, system dynamics. In addition, the motion equation used by the Kalman filter does not include disturbance torques with complex mathematical model because it is rather difficult to implement it to on-board computer. Usually, the influence of unaccounted perturbation on accuracy is investigated by simulation of the Kalman filter work. In this approach, computing takes a lot of time and its results are correct only with a certain probability.

Consider another approach to Kalman filter tuning. If the satellite attitude motion is sufficiently slow (or the measurement sampling frequency is high enough), we consider it as quasi-stationary. Consider the motion as quasi-stationary when the acting forces and the measurement model are nearly a constant in the time between consecutive measurements, i.e. $\Phi_k = \Phi \simeq const$, $H_k = H \simeq const$. For the discrete extended Kalman filter, one can calculate the covariance error matrix P_∞ after convergence. Hence, the filter performance quality after transient process is studied. The matrices at two consecutive steps should be equal:

$$P_\infty = P_k = P_{k-1}.$$

Therefore, the following matrix equation

$$P_\infty = [E - (\Phi P_\infty \Phi^T + Q) H^T [H(\Phi P_\infty \Phi^T + Q) H^T + R]^{-1} H](\Phi P_\infty \Phi^T + Q) \quad (5)$$

is valid. Note that all the matrices in this equation are considered to be constant. Taking into account that matrix P_∞ is symmetric, the considered nonlinear matrix equation can be rewritten as nonlinear equations with n unknown variables (i.e., the elements of matrix P_∞). These equations can be solved numerically, for example, by Newton's method.

2.3. Dependence of estimation accuracy on unaccounted perturbations

Let us consider how disturbances affect the estimation accuracy. The Kalman filter is developed for the linear motion and linear measurement models (or for the linearized ones) as follows

$$\mathbf{x}_{k+1} = \Phi_k \mathbf{x}_k + \mathbf{w}_k, \quad (6)$$

[1] SputniX Ltd technological satellites, the first one has been successfully launched on June 20, 2014 as a piggyback payload of the Dnepr rocket)