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Uniform rotations of tethered system connected to a moon surface

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ABSTRACT

We consider the problem of in-plane rotations of a space elevator with variable tether length attached to a surface of one of the primaries in a double system. The planet and its moon (or two asteroids) move about their center of mass in unperturbed elliptic Keplerian orbits. We discuss the possibilities to cause a prescribed motion of the system by changing the tether's length. Periodic solutions of the equation for the tether length control are studied using the method of small parameter. The stability of these solutions is studied numerically. The analysis shows that there exists a control law that implements tether rotations which are uniform with respect to true anomaly; one can indicate conditions when the above rotations are stable in the first approximation. These results can be used for the development of a planet elevator or a system for payload transportation to and from asteroid surface.

1. Introduction

Use of tethers for space transportation is currently discussed in several research groups and agencies. Tethers provide promising possibilities for orbital and attitude spacecraft control, distributed spacecraft missions, space debris removal, etc. [1,2]. One of the possible applications for space tethers is a space elevator. The project of building a space elevator at the Earth, though much awaited by numerous fans, still faces serious difficulties. Meanwhile, similar systems for Moon, Mars, or asteroid exploration look much more feasible.

Studies of spacecraft tethered to the Moon surface began long ago [3,4]. Several authors discuss possible applications of such structure for lunar exploration [1,2]. Recently some results on equilibria of such spacecraft and their stability have been obtained in [5–7].

Research on systems with variable mass distribution arises probably to [8–10] and to dissertation of V.A. Sarychev[1] (see also [11]). Analysis of the necessary conditions of stability for relative equilibria of a satellite with variable mass distribution has been done in [12], some of these results have been rediscovered in [13,14] (see also, e.g., [15]).

Various aspects of dynamics of orbital tethered system have been studied in [16–20]. Parametric analysis of orbiting tethers is performed in [21]. Dynamics of tethered systems in the vicinity of libration points is analyzed in [5,22]. Motions of a multi-tether system are considered in [23–30].

In [7,31–33] we consider dynamics of a tether anchored to the Moon surface and the possibilities to keep its orientation with respect to the Earth–Moon direction

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despite the eccentricity of the Moon orbit. A proper control of the tether's length can keep the fixed orientation of the tether for several modes of the system functioning. The results of [7,31,32] can be applied for other systems of two primaries, e.g., for planet's satellites or binary asteroids whenever the point of the tether's attachment maintains its orientation with respect to the other primary.

Meanwhile, the above results are only applicable for systems, where the moon's proper rotation is synchronized with its orbit motion so as the position of the tether's anchor is fixed with respect to the planet–moon direction. Relative rotation of the moon's surface requires that the tether follow the motion of the anchor. Here we examine the possibility to create a proper rotation of the tether via control of its length.

2. Posing the problem

Consider a system of two primaries, e.g., a planet E and a moon M, that move about their center of mass O in elliptic Keplerian orbits:

$$\left|\overrightarrow{OM}\right| = r = \frac{p_M}{1+e\cos\nu}, \quad \left|\overrightarrow{OE}\right| = \mu r = \frac{p_E}{1+e\cos\nu}, \quad \mu = \frac{m_M}{m_E}. \tag{1}$$

Here m_E and m_M are the masses of the planet and the moon respectively, p_E and p_M are parameters of their orbits, e is the eccentricity, and ν is the true anomaly. We assume that the sizes of the primaries are negligible. Fig. 1 shows the plane of the primaries' orbits π. The spacecraft of mass m is connected to the moon surface by a tether which length can be changed according to some control law. Only in-plane motions of the tether are considered; the tether orientation is described by angle φ (Fig. 1).

For in-plane motion of the tethered spacecraft its kinetic energy can be written as $T = (m/2)\left(\dot{x}_S^2+\dot{y}_S^2\right)$ or

$$T = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\nu}^2 + \dot{\ell}^2 + \ell^2(\dot{\nu}+\dot{\varphi})^2\right) + m\left(\dot{r}\dot{\ell} + r\ell\dot{\nu}(\dot{\nu}+\dot{\varphi})\right)\cos\varphi + m\left(\dot{\nu}(r\dot{\ell}-\ell\dot{r}) - \ell\dot{r}\dot{\varphi}\right)\sin\varphi. \tag{2}$$

Its potential energy is

$$U = -Gm\left(\frac{m_E}{r_E} + \frac{m_M}{\ell}\right). \tag{3}$$

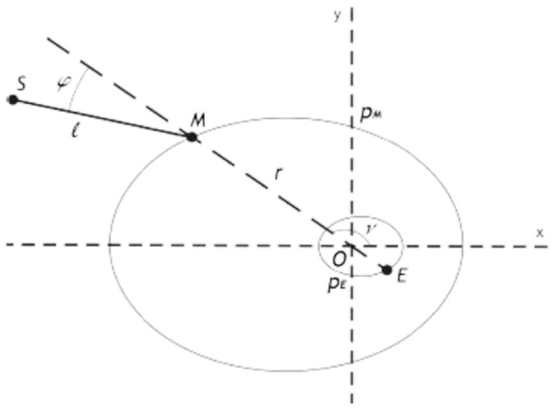

Fig. 1. Main notations.

Here G is the universal gravitational constant,

$$r_E = |SE| = \left(\rho^2 + 2\rho\ell\cos\varphi + \ell^2\right)^{1/2}$$

is the distance between the planet and the spacecraft, and $\rho = |EM| = (1+\mu)r$ is the distance between the planet and the moon. Assuming that ν, r, and ℓ are given as functions of time, one can write down the Lagrangian equations

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\varphi}} = \frac{\partial L}{\partial\varphi}, \quad L = T - U. \tag{4}$$

Introducing the true anomaly as a new independent variable and denoting by strike the respective derivative

$$\frac{d}{dt} = \dot{\nu}\frac{d}{d\nu} = \omega(1+e\cos\nu))^2\frac{d}{d\nu}, \tag{5}$$

where $\omega^2 = Gm_E/(1+\mu)^2 p_M^3$, the equation of motion can be rewritten as

$$\ell\varphi'' + 2\ell'(1+\varphi') - \frac{2e\ell(1+\varphi')\sin\nu}{1+e\cos\nu} + \frac{p_M\sin\varphi}{(1+e\cos\nu)^2}\left[1 - \frac{(1+\mu)^3 p_M^3}{f^{3/2}}\right] = 0,$$
$$f = \ell^2(1+e\cos\nu)^2 + (1+\mu)^2 p_M^2 + 2\ell(1+\mu)p_M\cos\varphi(1+e\cos\nu). \tag{6}$$

It is possible to consider Eqs. (6) from two different perspectives. In the framework of the direct problem, one can look for the system's motions that correspond to a specific variation of the tether length; in this case $\ell = \ell(\nu)$ is given and one has to study the second order ordinary differential equation for $\varphi(\nu)$. Considering the inverse problem, one can find the control law for the tether length $\ell = \ell(\nu)$ that results in a specific variation of the tether orientation; in this case $\varphi = \varphi(\nu)$ is defined beforehand and one has to find $\ell = \ell(\nu)$ analyzing the first-order differential equation (6). Here we examine the possibility to implement rotations of the tether, which are uniform with respect to the true anomaly. First we consider the inverse problem and find the control law $\ell = \ell(\nu)$ that causes such motions. Afterwards we analyze the direct problem and find the necessary conditions of stability for the above rotations. Similar problem regarding existence and stability of rotations of a tethered system has been studied in [34].

3. Forces in tether with variable length

Consider the Lagrangian

$$\Lambda(\varphi, \ell, \dot{\varphi}, \dot{\ell}) = T - U, \tag{7}$$

Here r and ν are given functions of time t. The equations of motion are

$$\frac{d}{dt}\frac{\partial\Lambda}{\partial\dot{\varphi}} = \frac{\partial\Lambda}{\partial\varphi}, \quad \frac{d}{dt}\frac{\partial\Lambda}{\partial\dot{\ell}} = \frac{\partial\Lambda}{\partial\ell} + F_\ell, \tag{8}$$

where F_ℓ is the generalized force correspondent to the coordinate ℓ. Since

$$\frac{\partial\Lambda}{\partial\dot{\ell}} = m\left(\dot{\ell} + \dot{r}\cos\varphi + \dot{\nu}r\sin\varphi\right),$$
$$\frac{\partial\Lambda}{\partial\ell} = m\left(\ell(\dot{\nu}+\dot{\varphi}) + r\dot{\nu}\cos\varphi - \dot{r}\sin\varphi\right)(\dot{\nu}+\dot{\varphi})$$