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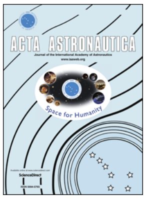

Flat-spin recovery of spinning satellites by an equatorial torque

Frank L. Janssens [a], Jozef C. van der Ha [b,*]

[a] Wilhelminastraat 29, 2201 KA Noordwijk, The Netherlands
[b] 5808 Bell Creek Road, Deming, WA 98244, USA

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ABSTRACT

The recovery from a flat-spin motion represents one of the most impressive practical applications in the field of spinning-satellite dynamics. The present paper presents flat-spin recovery maneuvers by means of a body-fixed torque within the plane perpendicular to the maximum principal axis of inertia. The conditions for a successful recovery are established. These are quite different from those obtained in the case when the torque is along the minimum axis of inertia where a minimum torque level is required for a successful recovery. If the torque component along the intermediate axis is negative, a recovery from a pure flat spin can be established for any torque magnitude. However, the time to recovery increases indefinitely when this torque component approaches zero. During the recovery maneuver, the angular velocity and angular momentum vectors become aligned with the minimum axis of inertia by turning over about 90° in the body frame. In inertial space, however, the angular momentum stays in the vicinity of its orientation before the start of the recovery.

1. Introduction

The problem of flat-spin recovery made its appearance when spinning satellites had to perform orbit injection maneuvers into their final mission orbit or trajectory after having been launched in a transfer or parking orbit. Therefore, spacecraft were equipped with a rocket motor and in most cases the spacecraft-motor configuration became prolate. Hence, it was spinning about the minimum axis of inertia. In the presence of dissipation (e.g., fuel slosh, vibrations) and in the absence of an active stabilization mechanism, the spacecraft would reorient itself and end up spinning about its maximum axis of inertia as illustrated in Fig. 1. We call this state '*pure flat spin*' or '*flat spin*' depending on the presence of nutation about the maximum axis. The re-establishment of a spin about the minimum axis is called '*flat-spin recovery*'.

One of the first papers about flat-spin recovery is by Barba et al. [1] (1973) and focuses on the SMS meteorological satellite. In the first part, a recovery procedure using a pure spin-up torque about the minimum axis is analyzed. For an asymmetric satellite they find that the torque must exceed a critical value to make the recovery possible. However, their derivation of the critical value uses the assumption of a symmetrical satellite. As a consequence, their critical value is unfortunately too low for an asymmetric satellite. Also they develop a recovery procedure using the available thrusters on SMS. The analysis is done by numerical simulations and illustrates very well the practical implementation of a recovery.

The model they use is a special case of the mathematical problem known as the self-excited rigid body (SERB) [2–5].

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[*] Corresponding author.
E-mail addresses: f.janssens@ziggo.nl (F.L. Janssens), jvdha@aol.com (J.C. van der Ha).

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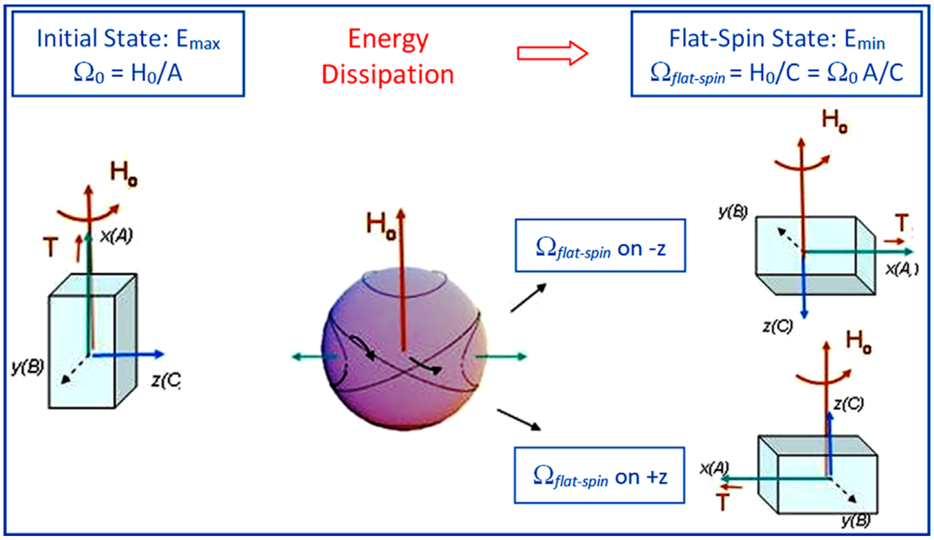

Fig. 1. Visualization of possible flat-spin attitude sequences.

This field of research deals with the dynamics of a spinning body subject to a torque with constant components in the body's coordinate frame. Progress in mathematical methods in this field can be useful for the design of efficient recovery procedures and other satellite dynamics applications as shown more recently by Longuski and Tsiotras [6].

Cronin (1978) [7] derives the correct value for the critical torque about the minimum axis of inertia of an asymmetric body for initial conditions of a pure flat-spin. Livneh and Wie [8,9] find the same result in modern terminology together with a complete discussion of torques on any of the principal axes.

As illustrated in Fig. 1, a transition into a flat spin may result in a positive or negative spin about the major axis of inertia. Rahn and Barba [10] show how the desired orientation of a spacecraft entering into a flat-spin can be achieved by two thruster impulses.

Recovery procedures based on a torque motor, as opposed to thrusters, have also been investigated [11,12]. In these cases, the SERB model is replaced by the dual-spin dynamical model.

In a previous paper [13] we present analytical results for the flat-spin recovery under a body-fixed torque pointing along the minor principal axis for arbitrary initial nutation conditions. We derive a solution in the form of a generalized pendulum equation for the increasing angular velocity along the torque axis in the body frame. Thus, the motion is similar to that of a pendulum which is either oscillating (no recovery) or revolving (recovery) with increasing angular velocity. The minimum torque level that guarantees a flat-spin recovery occurs precisely at the transition between these two cases. Also we found that the minimum required torque level for a recovery depends on the nutation phase angle. Approximate analytical results show the motion in inertial coordinates.

The pendulum-type solution follows from the existence of two first integrals. The first integration constant states that the amplitude of the rotational motion in the plane perpendicular to the torque remains constant. Ref. [13] presents explicit results for the decrease in the nutation angle after the recovery.

In this paper, we consider bodies with three different moments of inertia subjected to a more general torque acting within the plane normal the maximum inertia axis. In this case, only one first integral is available and no pendulum-like solution can be constructed. Thus, we study here only recovery strategies that start from a *pure* flat-spin situation, i.e. in the absence of nutation.

In the present case, the transition to the flat-spin recovery does not need to happen in the first revolution as was the case for a torque about the minimum-inertia axis in Ref. [13]. When the torque component on the intermediate axis is negative, the angular velocity along the major axis shows a secular decrease and the transition to a rotation about the minimum inertia axis will occur eventually. For a given torque magnitude, we can establish the optimum orientation of the torque that minimizes the time until the transition.

2. Dynamical equations of motion

The motion of an asymmetric rigid body under a constant body-fixed torque is described by the Euler equations [14]:

$$A\dot{\omega}_1 + (C-B)\omega_2\omega_3 = T_1 \tag{1a}$$

$$B\dot{\omega}_2 - (C-A)\omega_1\omega_3 = T_2 \tag{1b}$$

$$C\dot{\omega}_3 + (B-A)\omega_1\omega_2 = T_3 \tag{1c}$$

Here, the dot denotes the time-derivative, ω_j are the components of the rotation vector $\boldsymbol{\omega}$. The subscripts $j=1$, 2, 3 refer to the x, y, z principal body axes that are associated with the principal moments of inertia A, B, and C, respectively, and satisfy the following sequence:

$$A < B < C \tag{2}$$