Acta Astronautica 111 (2015) 270–282

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro

Switch points for highly eccentric orbits: Modelling the occurrences of sign changes in the rate of change of the eccentricity ☆

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ARTICLE INFO

Article history:
Received 17 May 2014
Received in revised form
16 February 2015
Accepted 23 February 2015
Available online 6 March 2015

Keywords:
Orbital mechanics
Lunisolar perturbations
Highly eccentric orbits

ABSTRACT

The lunar and solar gravitational perturbations coupled with the J_2 effect acting on Earth-orbiting satellites in critically inclined highly eccentric orbits cause several modes of oscillation in the eccentricity of the orbit. The sign changes in the slope of the eccentricity variations are labelled “switch points” and their development is explored in this paper. Using spherical trigonometry, the switch points can be determined in terms of the position and orbital elements of the perturbing third body (i.e., Moon or Sun) and the orbital elements of the satellite. Furthermore, the concept of switch point angles, used to identify when the switch points occur, is defined and discussed. As a result, the fundamental nature of the interaction between the gravitational perturbations of the third body and the orbit of the satellite is expanded upon. The switch points are also used to analyze the different modes of oscillation that occur in the eccentricity variations, whose periods can vary from two weeks to several years, and as a result cause significant variations in the perigee and apogee altitudes of the highly eccentric orbits. This study provides insight into the behaviour of the satellite's orbit under the lunar and solar perturbations in relation to the position of the Moon and Sun.

1. Introduction

The lunisolar perturbations acting on a satellite in a highly eccentric orbit (HEO) around the Earth cause large variations in the eccentricity which in turn significantly affect the perigee and apogee altitudes of the orbit and could potentially reduce the lifetime of the orbit [1]. While the effects of lunisolar perturbations on eccentric orbits have been studied in the past [1–3], this paper focuses entirely on their ability to significantly vary the eccentricity over time. The analysis presented here is valid for any type of HEO with a critical inclination and an argument of perigee set to 270°; however, this paper only studies the Molniya orbit. Along with the lunisolar perturbations, the most significant perturbation acting on HEOs is the effect from the equatorial bulging, or J_2. While J_2 does not directly contribute to secular or long-period variations in the eccentricity of HEOs, its effect on the right ascension of the ascending node (RAAN) of the satellite's orbit indirectly causes very long period oscillations of the eccentricity [4]. The perturbations from atmospheric drag and solar radiation pressure are not considered in this study since the selected baseline perigee altitude for the Molniya orbit is 1000 km and solar radiation pressure is not a primary perturbation [3].

Modelling of lunisolar perturbations acting on artificial satellites has been ongoing since the 1950s [5] and a plethora of methods exist for attempting to model their

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☆ Paper IAC-13-C1.9.9, presented at the 64th International Astronautical Congress in Beijing, China.

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http://dx.doi.org/10.1016/j.actaastro.2015.02.028
0094-5765/

Nomenclature

J_2	gravitational coefficient that models the equatorial bulging of the Earth
$\vec{N}$	ascending node vector
R_E	equatorial radius of the Earth
a	semi-major axis
e	eccentricity
$\vec{h}$	angular momentum vector
i	inclination
p	semi-latus rectum
r	orbital radius w.r.t. Earth
u	argument of latitude (AOL)
$\Delta\Omega$	relative RAAN between the satellite and the third body
Δi	relative inclination between the satellite and the third body
Σi	the sum of the orbital inclination of the satellite and the third body
Ω	right ascension of ascending node (RAAN)
α	the geocentric arc length between the satellite node vector and the third body position vector
β	the angle between the satellite plane and the plane defined by α, measured CW
γ	the angle between the equatorial plane and the plane defined by α, measured CW
δ	the angle between the α-plane and the β-plane along the orbit of the third body
μ	gravitational parameter
ϕ	geocentric arc length between the satellite and the third body
ω	argument of perigee (AOP)

Note: any element with subscript 3 refers the elements of a generic third body.

effects analytically. The authors implemented several of these methods in MATLAB and qualitatively compared them to both simulations performed using the software package Systems ToolKit (STK) [6] as well as Two-line Element data from actual satellite missions [7]. The results showed that the set of equations developed by Cook in 1962 [5] and those by Kaula also in 1962 [8] provide very accurate methods for modelling the lunisolar perturbations. However, the equations by Cook, which are averaged over the orbit period of the satellite, were found to be simpler to transform into a form suitable for studying the oscillations of the eccentricity.

In 1962, Smith published a paper which continued the analysis performed by Cook by performing numerical simulations of the lunar and solar perturbations [9]. In his paper, the author studied the conditions at which the rate of change of the eccentricity would change sign, which he referred to as 'turning points'. However, his analysis was limited to orbits which had an inclination of zero. Regardless of the assumptions, Smith's paper established that there is a unique relationship between the position of the third body (i.e., Moon or Sun) and the sign of the rate of change of the eccentricity.

This paper will introduce the concept of switch points,[1] which are defined as the occurrence of a sign change in the slope of the eccentricity for a highly eccentric orbit. Due to the additive effect of the separate perturbations of the Moon and Sun, multiple types of switch points can occur. It will be shown that there exist angles that relate the position and orbit of the Moon or Sun to the orbit of the satellite which can be used to predict when the switch points occur. The method to mathematically determine these angles, called switch point angles, will be described and discussed. Furthermore, it will be shown that the behaviour of these switch point angles is strongly affected by the difference in the RAAN of the satellite and third body and that it is through this relationship that the long-period oscillations of the eccentricity occur. This study not only provides a means for predicting switch points, which is important for orbital analysis, it also helps expand the knowledge of the fundamental nature of the effects of the lunisolar perturbations on highly eccentric orbits. Furthermore, it can be used as part of an orbital control strategy to reduce to required ΔV to maintain the eccentricity and the argument of perigee [10].

2. Types of oscillation modes and switch points

This section introduces the different modes of oscillation that occur in the eccentricity variations as well as classifies the types of switch points and provides the mathematical development of the switch point angles.

2.1. Variation of the eccentricity

As previously mentioned, the eccentricity of an HEO is subject to large variations due to the coupling of lunisolar perturbations and the secular J_2 effects on the RAAN and Argument of Perigee (AOP). These variations are modelled using the six ODEs provided by Cook and are implemented into a MATLAB script which numerically integrates the equations using a 4th order variable-step Runge–Kutta–Felhberg numerical integrator [5]. All of the figures presented in this paper were created with the orbit data computed in MATLAB. For the reference Molniya orbit studied in this paper, the following initial conditions were used: 11.96 h orbit period, 1000 km perigee altitude, inclination of 63.4°, RAAN of 63°,[2] and an AOP of 270°. The eccentricity variations of a 7300 day simulation of the HEO are shown in Fig. 1. Two modes of oscillations can be observed in this plot: long-period oscillations of approximately 7 years and medium-period oscillations of approximately 0.5 years. The long-period oscillations are caused

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[1] The terms 'switch point' and 'turning point' are synonymous.

[2] The J2000 Earth-centered coordinate system is used to define the RAAN and other orbital angles defined in this paper.