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# Scaled control moment gyroscope dynamics effects on performance

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ABSTRACT

The majority of the literature that discusses the dynamics of control moment gyroscopes (CMG) contains formulations that are not derived from first principles and make simplifying assumptions early in the derivation, possibly neglecting important contributions. For small satellites, additional dynamics that are no longer negligible are shown to cause an increase in torque error and loss of torque amplification. The goal of the analysis presented here is to provide the reader with a complete and general analytical derivation of the equations for dynamics of a spacecraft with $n$-CMG and to discuss the performance degradation imposed to CMG actuators when scaling them for small satellites. The paper first derives the equations of motion from first principles for a very general case of a spacecraft with $n$-CMG. Each contribution of the dynamics is described with its effect on the performance of CMG and its significance on scaled CMG performance is addressed. It is shown analytically and verified numerically, that CMG do not scale properly with performance and care must be taken in their design to trade performance, size, mass, and power when reducing their scale.



## 1. Introduction

Control moment gyroscopes (CMG) are momentum control actuators used for precise attitude control that provide higher torque per unit power, mass, and volume than reaction wheel assemblies (RWA). Much of the literature discusses the benefits and complexities of single-gimbal CMG over RWA due to their property of torque amplification and their inherent geometric singularities [1,14,7]. However, very little analysis has been published on describing efficiency in terms of torque amplification properties and accuracy for their use in operation. Only some work exists that presents a detailed analysis of their dynamics equations of motion [21]. Even less prevalent is how previously neglected dynamics for small CMG systems affects CMG and how this performance scales with their use in non-traditional applications (e.g., space-robotics [17]). When considering smaller satellites, the trade-off in performance induced from scaling CMG needs to be understood by the satellite designer.

Considering the dynamics and control of spacecraft with CMG actuators, an elegant matrix-form dynamics formulation was presented in Schaub et al. [21] for a spacecraft with variable speed CMG. A more specific subset of the dynamics formulation for CMG in Schaub et al. [21] is shown in Sun et al. [23], which considered the effect of gimbal-wheel assembly inertia as an attitude disturbance to the spacecraft–CMG system. Here, an even more general set of the equations of motion for a spacecraft with $n$-CMG is derived and is presented. Next, perturbative terms to the angular momentum that are not an effect of the CMG design are neglected. Finally, numerical examples are given to follow the analytics and

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show the degradative effect of scaling CMG actuators on their accuracy and torque amplification.

The paper is organized as follows: Section 2 derives the complete equations of motion for a spacecraft containing $n$ CMG; Section 3 shows analytically, and through numerical simulation, the degradative effect of scaling CMG on steering algorithm performance; Section 4 derives the full equations for torque amplification of a single CMG and reduces them to a form that is amenable to describing the degradative effect of scaling CMG on their performance in terms of torque amplification and verifies this effect numerically. The paper ends with the conclusion of the presented analysis.

## 2. Rotational kinematics and kinetics of a spacecraft with CMG

This section provides a derivation of the kinematics and kinetics of a spacecraft with $n$ CMG from first principles. First, the angular momentum from each moving component of the spacecraft–CMG system is derived. Next, the angular momenta will be differentiated with respect to time, providing the general equations of motion through Euler's Second Law. From these equations, contributions of specific terms to angular momentum perturbations are identified and those that are assumed irrelevant to the performance degradation of the CMG in attitude control performance are neglected.

The notation used for inertia dyadics and angular momentum vectors $\mathbf{x}_{()}^{()}$ contain superscripts and subscripts where the superscript stands for the body which the inertia and angular momentum contribution are defined and the subscript indicates the point about which they are taken. The position vectors in this formulation have only a subscript which refers to the position of their corresponding point. Any point designated with a $C_{()}$ is the CM of that particular component (e.g., $C_{Gi}$ in Fig. 2 refers to the CM of the $i$th gimbal). Consult the nomenclature in the appendix for further details on notation.

First, let us consider a spacecraft with a single CMG as shown in Fig. 1. In Fig. 1, we are not assuming that the center of mass (CM) of the gimbal, rotor, nor spacecraft body is fixed in the spacecraft body frame $\mathcal{F}_B$. Therefore, we consider the CM positions of the spacecraft with respect to $\mathcal{F}_B$ and that of the gimbal and rotor with respect to their local reference frames $\mathcal{F}_{G_i}$ and $\mathcal{F}_{W_i}$ for a CMG with origin $p_i$ at a position, $\mathbf{r}_{CMGi}$ from the point $A$, as shown in Fig. 2. The point $p_i$ is the origin of the $i$th CMG body, assumed to be fixed point in the body frame, and located between the intersection of the $i$th gimbal and rotor axes.

The geometry for a system consisting of a spacecraft with one single-gimbal CMG (SGCMG) is illustrated in Figs. 1 and 2. The total angular momentum, $\mathbf{H}_A^S$ about a point $A$, of the system $S$, consisting of a spacecraft with $n$-CMG, is the sum of component momenta contributions

$$\mathbf{H}_A^S = \mathbf{h}_A^B + \sum_{i=1}^{n} \mathbf{h}_A^{CMGi} = \mathbf{h}_A^B + \sum_{i=1}^{n} (\mathbf{h}_A^{Gi} + \mathbf{h}_A^{Wi}), \tag{1}$$

where $\mathbf{h}_A^B$, $\mathbf{h}_A^{Gi}$, and $\mathbf{h}_A^{Wi}$ denote the component angular momentum contributions from the spacecraft bus

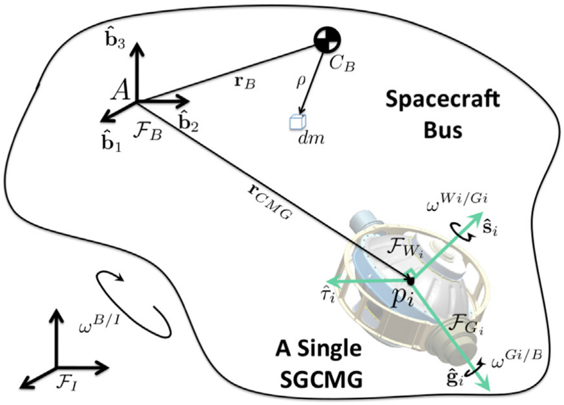


**Fig. 1.** Spacecraft with a single CMG (CMG Figure Courtesy of Brian Hamilton, Honeywell Defense and Space, Glendale, AZ).

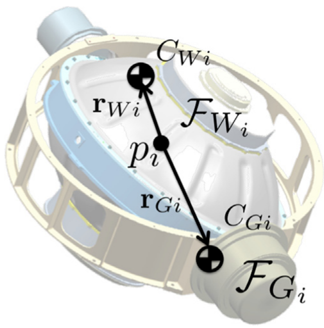


**Fig. 2.** CMG rotor and gimbal CM offsets (CMG Figure Courtesy of Brian Hamilton, Honeywell Defense and Space, Glendale, AZ).

(carrier body), CMG gimbals, and rotors about $A$, respectively.

### 2.1. Spacecraft angular momentum

If the CM of the spacecraft $n$-CMG system is not fixed with respect to the spacecraft body frame, the angular momentum may be taken about a fixed point on this frame first and then transferred to the CM later. Mass integrals for angular momentum have the form $\int_A \mathbf{r} \times \mathbf{v}$ for a body $A$[1] where $\mathbf{r}$ is the position vector from the point about where the angular momentum is taken and $\mathbf{v}$ is the inertial translational velocity. Therefore, as shown in Fig. 1, the carrier body $B$ (i.e., spacecraft body excluding CMG) angular momentum, $\mathbf{h}_A^B$ of the spacecraft about a fixed point A, in the spacecraft body frame, is represented by a mass integral about the spacecraft body

$$\mathbf{h}_A^B = \int_B (\mathbf{r}_B + \boldsymbol{\rho}) \times [\mathbf{v}_A + \boldsymbol{\omega}^{B/I} \times (\mathbf{r}_B + \boldsymbol{\rho})]\, dm, \tag{2}$$

where $\mathbf{r}_B$ is the position of the spacecraft CM, $C_B$ relative to point $A$, $\boldsymbol{\rho}$ is the position vector of any differential mass element relative to $C_B$, $\mathbf{v}_A$ is the translational velocity vector of the point $A$, and $\boldsymbol{\omega}^{B/I}$ is the angular velocity

[1] A body and a reference frame may be different where the reference frame is a set of at least three non-collinear points fixed in the body.