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Performance evaluation of the inverse dynamics method for optimal spacecraft reorientation

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ABSTRACT

This paper investigates the application of the inverse dynamics in the virtual domain method to Euler angles, quaternions, and modified Rodrigues parameters for rapid optimal attitude trajectory generation for spacecraft reorientation maneuvers. The impact of the virtual domain and attitude representation is numerically investigated for both minimum time and minimum energy problems. Owing to the nature of the inverse dynamics method, it yields sub-optimal solutions for minimum time problems. Furthermore, the virtual domain improves the optimality of the solution, but at the cost of more computational time. The attitude representation also affects solution quality and computational speed. For minimum energy problems, the optimal solution can be obtained without the virtual domain with any considered attitude representation.

1. Introduction

The reorientation of a spacecraft is a common task in most space missions. For instance, slew maneuvers are required for targeting imaging equipment and sensors, orienting antenna towards Earth or reorienting the spacecraft for solar power absorption. However, in particular applications the spacecraft is required to reorient while minimizing a certain mission parameter, such as maneuver duration or fuel expenditure.

Rapid retargeting maneuvers, also called time-optimal reorientation maneuvers, are required by Earth imaging satellite in order to increase mission effectiveness. Rapid retargeting capability increases the image collection capacity during a given observation window, which is a key aspect for commercial applications or climate and natural disaster monitoring [1]. Moreover, pointing the entire spacecraft rather than sweep the imaging system improves the resolution of the image [2]. Military space missions may also require agile reorientation capabilities. For instance, the TacSat-3 was developed to demonstrate responsive delivery of information to operational military users [3]. The key feature required by the satellite was the capability to rapidly change its attitude due to the limited operational window for mission acquisition, tasking and information delivery. Finally, agile attitude maneuvers are also required by tracking satellites for pointing moving ground targets [2].

Current onboard control systems execute the maneuver by steering about a spacecraft eigenaxis since it represents the shortest angular path between two orientations. However, Karpenko et al. demonstrated with the TRACE spacecraft that this procedure is not time-optimal, and onboard control systems which optimize the maneuver are required [4].

The optimal reorientation of a spacecraft has been studied theoretically for both minimum time and minimum energy problems [5–7]. In particular, Bilimoria and Wie investigated the time optimal rest to rest reorientation of a symmetric spacecraft, showing that bang–bang control is optimal and the resulting motion has a significant nutational component [6].

Recently, the problem has been investigated numerically using two different direct optimization approaches: pseudospectral methods and inverse dynamics. Pseudospectral

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methods are based on the numerical integration of the differential equations of motion and provide accurate solutions, but may converge slowly [8,9]. On the contrary, in inverse dynamics the trajectory is approximated by analytical functions. In most cases the solution is sub-optimal, but computational speed is high due to the reduced number of variable parameters. Therefore, the inverse dynamics approach is suitable for onboard applications. Louembert et al. first applied the inverse dynamics method to the optimal spacecraft reorientation problem using B-Splines to represent the modified Rodrigues parameters [10]. Recently, Boyarko et al. proposed a rapid attitude trajectory generation method based on the inverse dynamics in the virtual domain (IDVD) [11]. Here the quaternion components are approximated by polynomials defined in an abstract argument (virtual domain). Finally, Yakimenko applied IDVD to Euler angles [12]. However, no performance evaluation of the IDVD method has been conducted.

The objective of the present paper is to investigate the performance of the inverse dynamics method for rapid optimal spacecraft attitude trajectory generation by evaluating the effects of the attitude representation and usage of virtual domain on solution quality and computational speed of the algorithm. To evaluate the impact of the virtual domain, the attitude trajectory is approximated in both time and virtual domains using analogous polynomial functions. To evaluate the impact of the attitude representation, the IDVD method is applied, both in time and virtual domains, to Euler angles, quaternions, and modified Rodrigues parameters (MRP). An ideal scenario taken from Bilimoria and Wie is analyzed for both minimum time and minimum energy problems. Additional scenarios generated with Monte Carlo simulation are investigated for the minimum time problem.

The present paper is organized as follows: the first section describes the optimal spacecraft reorientation problem and introduces the inverse dynamics optimization approach, including IDVD. The application of IDVD to the Euler angles, quaternion and MRP is then described. Numerical experiments and conclusions are reported in the last sections of the paper.

2. Problem formulation and optimization approach

This section introduces the optimal spacecraft reorientation problem and the IDVD method for solving optimal control problems. The spacecraft is assumed rigid body.

2.1. Rotational motion of the spacecraft and optimization problem formulation

The dynamics of the rotational motion of a spacecraft is governed by Euler's equations, which in scalar form with the angular velocity $\boldsymbol{\omega}=[\omega_x, \omega_y, \omega_z]^T$ and the inertial tensor $\boldsymbol{I}=\text{diag}([I_{xx}, I_{yy}, I_{zz}])$ referenced to the body-fixed principal axes can be expressed as [13]

$$\begin{cases} \dot{\omega}_x = \frac{(I_{yy}-I_{zz})\omega_z\omega_y + T_x}{I_{xx}} \\ \dot{\omega}_y = \frac{(I_{zz}-I_{xx})\omega_z\omega_x + T_y}{I_{yy}} \\ \dot{\omega}_z = \frac{(I_{xx}-I_{yy})\omega_y\omega_x + T_z}{I_{zz}} \end{cases} \tag{1}$$

In Eq. (1) $\mathbf{T}=[T_x, T_y, T_z]^T$ represents the component of the bounded external torque vector referenced to the body frame.

The kinematics of the rotational motion of a spacecraft can be described using different attitude representations [14]. In terms of quaternion $\boldsymbol{q}=[q_1,q_2,q_3,q_4]^T$, the kinematic differential equation is given by

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \frac{1}{2}\boldsymbol{q} \otimes \boldsymbol{\Omega}, \tag{2}$$

where $\mathbf{\Omega}=[\omega_x, \omega_y, \omega_z, 0]^T$ and the symbol $\otimes$ denotes the (Hamiltonian) product between quaternions [14]. In terms of Euler angles $\boldsymbol{\theta}=[\theta_1, \theta_2, \theta_3]^T$ of the rotational sequence 1–2–3, the kinematic differential equation is

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \frac{1}{\cos\theta_2}\begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3\cos\theta_2 & \cos\theta_3\cos\theta_2 & 0 \\ -\cos\theta_3\sin\theta_2 & \sin\theta_3\sin\theta_2 & 1 \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}. \tag{3}$$

Finally, using the MRP $\boldsymbol{\sigma}=[\sigma_1, \sigma_2, \sigma_3]^T$, the kinematics of the rotational motion can be described by the following equation

$$\begin{bmatrix} \dot{\sigma}_1 \\ \dot{\sigma}_2 \\ \dot{\sigma}_3 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1+\sigma_1^2-\sigma_2^2-\sigma_3^2 & 2(\sigma_1\sigma_2-\sigma_3) & 2(\sigma_1\sigma_3+\sigma_2) \\ 2(\sigma_1\sigma_2+\sigma_3) & 1-\sigma_1^2+\sigma_2^2-\sigma_3^2 & 2(\sigma_2\sigma_3-\sigma_1) \\ 2(\sigma_1\sigma_3-\sigma_2) & 2(\sigma_2\sigma_3+\sigma_1) & 1-\sigma_1^2-\sigma_2^2+\sigma_3^2 \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$
$$= \frac{1}{4}\boldsymbol{B}(\boldsymbol{\sigma})\boldsymbol{\omega}. \tag{4}$$

The optimal spacecraft reorientation problem consists of minimizing a (given) cost function J by finding the optimal control vector $\boldsymbol{T}$, subjected to the control constraints $\boldsymbol{T}_{min} \le \boldsymbol{T} \le \boldsymbol{T}_{max}$, that brings the system described by Eq. (1) and Eqs. (2)–(4) from an initial state of angular velocity $\boldsymbol{\omega}_0$ and attitude $\mathbf{q}_0$, $\boldsymbol{\theta}_0$ or $\boldsymbol{\sigma}_0$ to a final state of angular velocity $\boldsymbol{\omega}_F$ and attitude $\boldsymbol{q}_F$, $\boldsymbol{\theta}_F$ or $\boldsymbol{\sigma}_F$. The cost function J is defined as

$$J = \int_{t_0}^{t_F} \mathrm{d}t \tag{5}$$

for minimum time maneuvers and

$$J = \frac{1}{2}\int_{t_0}^{t_F} (T_x^2+T_y^2+T_z^2)\,\mathrm{d}t \tag{6}$$

for minimum quadratic control (or energy) expenditure.

2.2. The inverse dynamics and inverse dynamics in the virtual domain methods

In the inverse dynamics approach for rapid trajectory generation the optimal control problem is converted into an equivalent nonlinear programming problem by describing the trajectory components with a set of polynomial functions defined in the time domain. The cost is then minimized through the optimization of the polynomial coefficients.

The inverse dynamics in the virtual domain method (IDVD) follows the inverse dynamics approach by defining the polynomials for trajectory representation in a virtual