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A new approach to trajectory optimization based on direct transcription and differential flatness

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ABSTRACT

The objective of the present paper is to introduce a reliable method to produce an optimal trajectory in the presence of all limitations and constraints. Direct transcription, has been employed to convert the trajectory optimization problem into nonlinear programming problem via discretizing the profile of state and control parameters and solving for a constrained problem. Differential flatness as a complementary theory leads to model the optimization problem in a lowered dimensional space through defining flat variables. Several curvilinear functions have been used to approximate flat variables and have their own benefits and disadvantages. Accuracy, complexity and number of needed points are examples of related issues. A new approach is developed based on an indirect approximation of flat variables, which leads to decrease the optimization variables and computational costs while preserving the needed accuracy. The proposed method deals with a 3rd order approximation of flat variables via integrating linear function of the acceleration profile. The new method is implemented on the terminal area energy management phase of a reusable launch vehicle. Test results show that the suggested method, as compared with other conventional methods, requires lower computational efforts in cases of the number of iterations and function evaluations, while providing a more accurate optimal solution.

1. Introduction

Optimal trajectory generation is a vast field of research among the engineering fields including aerospace engineering. A wide variety of techniques have been employed to solve trajectory optimization problems. One set of techniques are those which are developed, based upon battle-field geometry and such line of sight angle as proportional navigation [1]. In these methods the goal is only to reach the terminal point and to make the impact error as small as possible. Some other techniques are those resulting from the solution of optimal control problem via defining the co-state variables in addition to state variables and solving for a Two-Point Boundary Value Problem (TPBVP). The latter is also called an indirect method of optimization which has its own complexities on providing initial conditions of co-state variables and evaluating analytical calculations for each case example of trajectory optimization. Because of such issues, this method has been used only for some case examples as one of them being reviewed by Petropoulos and Sims [2]. Linear Quadratic Regulator (LQR) is an advanced method of solving TPBVP, considering the linear relation between states and co-states while still analytical calculations needed if there is a need to change the objectives of the problem [3]. Also, shooting methods are widely used in solving TPBVP problems.

Direct methods as the third set of trajectory optimization techniques are developed based on avoiding the

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challenges in solving TPBVP, via converting the trajectory optimization problem into a Nonlinear Programming problem (NLP). This work is done through parameterization of problem states time history and subdivided into several specific time intervals. The optimization problem can then be solved by use of NLP techniques as Sequential Quadratic Programming (SQP) [4,5]. A very useful benefit of direct Optimization is the relief in defining any objectives and constraints with no need of making analytical calculations.

In direct methods, state and control variables are usually directly approximated, by use of such curvilinear functions as Cubic splines and hermit polynomials [6,7], B-splines [8], Bezier [9] and also Pseudo-spectral methods, [10,11,12]. NLP techniques are then employed to optimize the approximated function parameters, and subsequently make the trajectory not violate the path and terminal constraints. Some software packages are also developed as based on such direct optimization methods as DIDO [13].

Another beneficial technique recently developed to solve trajectory optimization problem is differential flatness [14,15,16,17] in which the dimensions of the problem are decreased to have more computational performances.

With recent advances in computational techniques, some new approaches in optimization have emerged so that in addition to off-line trajectory optimization, the online methods are also under consideration [18].

The objective of this paper is to introduce a reliable method to produce an optimal trajectory as based on a combination of the direct optimization method and differential flatness theory. Approximation of flat variables has been conducted via a new approach and with some differences as compared with the conventional methods, including B-splines or Bezier curves. Finally the developed method has been tested on a re-entry trajectory optimization problem.

The remainder of the paper is organized as follows; in Section 2, the knowledge of optimization problem using nonlinear programming and direct transcription is investigated. Meanwhile differential flatness is taken into consideration and some methods of approximating flat variables are briefly reviewed. In Section 3 a new approach to approximate flat variables is introduced and in Section 4 the case example of re-entry optimization problem implemented on space-shuttle configuration model enclosed with re-entry dynamics is described as well. Finally the results of re-entry trajectory optimization are presented and discussed.

2. Optimal nonlinear trajectory generation

The governing rules of an optimal control problem, direct transcription and the differential flatness will be presented within the related section. Also approximation methods will be reviewed briefly.

2.1. Optimal control problem

Optimal control problem can be concluded as: choose the control signal $u(t)$ to minimize the cost,

$$J=\varphi_{t_0,t_f}(x(t_{t_0,t_f}),u(t_{t_0,t_f}))+\int_{t_0}^{t_f} L(x(t),u(t))dt \quad (1)$$

where L is a nonlinear function and φ_{t_0,t_f}, vector of initial and final, subject to state equation as follows:

$$\begin{aligned}&\dot{x}=f(x(t),u(t))\\ &lb\le x\le ub\\ &t\in[t_0,t_f]\end{aligned} \quad (2)$$

boundary and path constraint

$$\psi(x(t),u(t),t)=0,\quad t=t_0,t_f \quad (3)$$

$$C_l\le C(x(t),u(t),t)\le C_u \quad (4)$$

2.2. Direct transcription

To solve an optimal control problem, some related techniques have been developed, one of which is direct transcription. The main idea of direct transcription is to convert an optimal control problem into a parameterization one and to create a finite number of variables known as Nonlinear Programming (NLP) variables [19]. The first step is to subdivide continuous problem dynamics $[t_0,t_f]$ into a discrete profile via defining n sequences of smaller time domains, which is called phases. The boundaries of each phase end up with a point named node.

$$t_0=t_1<t_2<\ldots<t_n=t_f \quad (5)$$

Thus, each phase length can be shown as $h_k=t_{k+1}-t_k$ and is sequential for most applications, that is, ${t_{k+1}}^{ph_n}={t_k}^{ph_{n+1}}$.

A classical approach to direct transcription is direct collocation which for the first time outlined by Hargraves and Paris [7]. In direct collocation the state and control histories are represented by Hermit polynomials within each phase. A cubic polynomial could then be used to approximate the states and controls:

$$x=C_0+C_1t+C_2t^2+C_3t^3 \quad (6)$$

Coefficients (C_0 to C_3) can be calculated by use of initial conditions (state values and state derivatives) at the boundaries of each collocation interval. The dynamics of the system are then defined at the midpoint by imposing an equality constraint resulted from forcing equality of slope of x and dynamics of the system.

$$\begin{aligned}&\zeta=x_{k+1}-x_k-\tfrac{h_k}{6}(f_k+4\overline{f}_{k+1}+f_{k+1})=0\\ &\overline{f}=f(\overline{y}_{k+1},\overline{u}_{k+1},t_k+\frac{h_k}{2})\\ &\overline{y}_{k+1}=\frac{1}{2}(y_k+y_{k+1})+\frac{h_k}{8}(f_k-f_{k+1})\end{aligned} \quad (7)$$

where ζ is called a defect with $f_k,\overline{f}_{k+1},f_{k+1}$ representing the values of dynamic system at the first, mid, and the end points of an interval respectively.

Regarding (7), differential equations will be replaced by a finite set of defect constraints. As a result of direct collocation; the optimal control constraints (3) and (4), and the integration process, are replaced by the nonlinear programming constraints,

$$\begin{aligned}&g(x)=[\zeta_1,\zeta_2,\ldots,\zeta_{n-1},\psi_I,\psi_F,C_1,C_2,\ldots,C_m]\\ &g_L\le g(x)\le g_U\end{aligned} \quad (8)$$