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Nonlinearity analysis of measurement model for vision-based optical navigation system

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ABSTRACT

In the autonomous optical navigation system based on line-of-sight vector observation, nonlinearity of measurement model is highly correlated with the navigation performance. By quantitatively calculating the degree of nonlinearity of the focal plane model and the unit vector model, this paper focuses on determining which optical measurement model performs better. Firstly, measurement equations and measurement noise statistics of these two line-of-sight measurement models are established based on perspective projection co-linearity equation. Then the nonlinear effects of measurement model on the filter performance are analyzed within the framework of the Extended Kalman filter, also the degrees of nonlinearity of two measurement models are compared using the curvature measure theory from differential geometry. Finally, a simulation of star-tracker-based attitude determination is presented to confirm the superiority of the unit vector measurement model. Simulation results show that the magnitude of curvature nonlinearity measurement is consistent with the filter performance, and the unit vector measurement model yields higher estimation precision and faster convergence properties.

1. Introduction

As the development of vision sensors, autonomous optical navigation and attitude determination system based on line-of-sight vector measurement have attracted much attention in space missions, especially in interplanetary exploration missions [1–3]. The optical navigation technology was first implemented to improve the precision of state estimation in Voyager mission during its encounters with Jupiter, Saturn, Uranus, etc [4,5]. The Deep Space 1 (DS1) mission was the first mission that successfully validated the autonomous optical navigation system [6,7]. NASA's Mars Exploration Rover mission including Spirit and Opportunity rovers achieved the first successful application of optical navigation techniques in Mars landing mission [8]. The dependence of navigation system on optical sensor motivates the need for high precision measurement sensors. In the nonlinear setting, when the prior state estimation errors are large, simply neglecting the nonlinear high-order terms in the filter process might lead to filter divergence or convergence to a wrong estimate value [9]. This situation will become more acute if the measurement function nonlinearity is comparable to the measurement noise [10]. Therefore, the nonlinearity of measurement model plays an essential role in optical sensors, especially the ones whose measurements are of very high precision.

The Extended Kalman filter (EKF) has been widely used in navigation systems during the past few years. As an approximate nonlinear estimator, EKF assumes that system nonlinear models can be well approximated by linear models using Taylor series expansions with respect to the

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current predicted state estimate. When the measurement model is highly nonlinear near the state estimate, the linear approximation of the EKF may perform poorly. To obtain optimal filter performance, it is better to select a measurement model with low degree of nonlinearity and compensate for nonlinear high-order terms in the meantime. Therefore in nonlinear estimate problem, the ability to quantitatively measure the nonlinearity of the models is of great importance. Some advanced nonlinear filter methods including the particle filter (PF) and the unscented Kalman filter (UKF) can be used to deal with different degrees of nonlinearity in navigation system. However the real-time processing requirement of space missions makes the PF unsuitable for engineering application [11]. The filtering performance of the UKF relies on online tuning of the filter parameters, so acceptable filtering results may not always be achieved in practical application [12].

Optical sensors observation models can be described by the focal plane measurement model and the unit vector measurement model [13]. For focal plane model, which is the basic measurement type of the optical sensors, the measurement equation is given by the well-known collinearity equation associated directly with actual physical observation quantities and true measurement noises. The collinearity equation can also be cast in the unit vector form by nonlinear transformation. The QUEST measurement model [14] further simplifies the measurement covariance matrix, and has been widely used in navigation systems. Cheng [15] showed that by using the unit vector model instead of the focal plane model can the better navigation result be achievable. However, the conclusion was proposed based on single case simulation, leading to the lacks of theoretical analysis and generality in application.

Existing measuring methods of nonlinearity mainly fall into two classes. The first one measures the deviation of nonlinear function from its best linear approximation model [16,17]. The other method is based on the nonlinear curvature measure theory from differential geometry, which has been applied to target tracking problems [18–20]. Since the latter method has clearer geometric and physical meaning, this paper follows the method developed by Bates and Watts [21] and uses it to analyze the degree of nonlinearity of optical measurement models. The effects of two measurement models on estimation accuracy are compared in detail within the application framework of star-tracker-based high-precision attitude determination system. Finally, simulation results are presented to verify the theoretical analysis results.

2. Attitude estimation algorithm

In this section, a general problem of spacecraft attitude estimation is proposed. We first utilize the unit quaternion as attitude representation parameter. The collinearity equations are briefly summarized, followed by the covariance model for the focal-plane measurement model and the unit vector measurement model. Then we analyze the effects of the measurement model nonlinearity on the estimation algorithm in detail.

2.1. Attitude kinematics and gyro model

As the globally non-singular attitude representation of the lowest dimension, the quaternion has been widely used in attitude estimation [22]. A typical quaternion can be defined as

$$\boldsymbol{q}=\begin{bmatrix}\boldsymbol{p}\\ q_4\end{bmatrix}=\begin{bmatrix}\boldsymbol{e}\sin(\vartheta/2)\\ \cos(\vartheta/2)\end{bmatrix} \tag{1}$$

where $\boldsymbol{e}$ is the unit Euler axis, ϑ is the angle of rotation, and $\boldsymbol{p}=[q_1,q_2,q_3]^{\mathrm{T}}$ is the quaternion vector, respectively. Since the four-dimensional unit quaternion is used to describe three dimensional attitude parameters, the quaternion is subject to unit norm constraint given by

$$|\boldsymbol{q}|^2=|\boldsymbol{p}|^2+q_4^2=1$$

The associated attitude rotation matrix is

$$\begin{aligned}\mathbf{A}(\boldsymbol{q})&=(q_4^2-\boldsymbol{p}^{\mathrm{T}}\boldsymbol{p})\boldsymbol{I}_{3\times3}+2\boldsymbol{p}\boldsymbol{p}^{\mathrm{T}}-2q_4[\boldsymbol{p}\times]\\&=\boldsymbol{\Xi}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{\Psi}(\boldsymbol{q})\end{aligned} \tag{2}$$

where

$$\boldsymbol{\Xi}(\boldsymbol{q})=\begin{bmatrix}q_4\boldsymbol{I}_{3\times3}+[\boldsymbol{p}\times]\\ -\boldsymbol{p}^{\mathrm{T}}\end{bmatrix}$$

$$\boldsymbol{\Psi}(\boldsymbol{q})=\begin{bmatrix}q_4\boldsymbol{I}_{3\times3}-[\boldsymbol{p}\times]\\ -\boldsymbol{p}^{\mathrm{T}}\end{bmatrix}$$

where $[\boldsymbol{p}\times]$ is the cross-product matrix defined as

$$[\boldsymbol{p}\times]=\begin{bmatrix}0&-q_3&q_2\\ q_3&0&-q_1\\ -q_2&q_1&0\end{bmatrix} \tag{3}$$

The quaternion product of $\boldsymbol{q}'$ and $\boldsymbol{q}$ is defined in the same order as the attitude matrix multiplication.

$$\boldsymbol{q}'\otimes\boldsymbol{q}=\begin{bmatrix}\boldsymbol{\Psi}(\boldsymbol{q}') & \boldsymbol{q}'\end{bmatrix}\boldsymbol{q}=\begin{bmatrix}\boldsymbol{\Xi}(\boldsymbol{q}) & \boldsymbol{q}\end{bmatrix}\boldsymbol{q}'' \tag{4}$$

The rotational kinematics equation for quaternion is given by

$$\dot{\boldsymbol{q}}=\frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{\omega})\boldsymbol{q}=\frac{1}{2}\boldsymbol{\Xi}(\boldsymbol{q})\boldsymbol{\omega} \tag{5}$$

where $\boldsymbol{\omega}$ is the angular velocity vector and

$$\boldsymbol{\Omega}(\boldsymbol{\omega})=\begin{bmatrix}-[\boldsymbol{\omega}\times] & \boldsymbol{\omega}\\ -\boldsymbol{\omega}^{\mathrm{T}} & 0\end{bmatrix} \tag{6}$$

We can see that the kinematic equation for quaternion in Eq. (5) is linear and successive rotation can be achieved by quaternion composition.

Although gyro-less attitude determination methods can incorporate attitude dynamics model to estimate angular rate, the filter parameters need to be adjusted repeatedly to satisfy the filter precision requirement due to the inaccuracy of the attitude dynamic equations. Therefore the angular velocity vector is typically measured by the rate integrating gyro. The mathematical model for such sensor is given by

$$\left.\begin{aligned}\tilde{\boldsymbol{\omega}}_{rg}&=\boldsymbol{\omega}+\boldsymbol{\omega}_{bias}+\boldsymbol{w}_a\\ \dot{\boldsymbol{\omega}}_{bias}&=\boldsymbol{w}_b\end{aligned}\right\} \tag{7}$$