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Attitude control with active actuator saturation prevention

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ABSTRACT

Spacecraft attitude control in the presence of actuator saturation is considered. The attitude controller developed has two components: a proportional component and an angular velocity component. The proportional control has a special form that depends on the attitude parameterization. The angular velocity control is realized by a strictly positive real system with its own input nonlinearity. The strictly positive real system can filter noise in the angular velocity measurement. With this control architecture the torques applied to the body are guaranteed to be below a predetermined value, thus preventing saturation of the actuators. The closed-loop equilibrium point corresponding to the desired attitude is shown to be asymptotically stable. Additionally, the control law does not require specific knowledge of the body's inertia properties, and is therefore robust to such modelling errors.

1. Introduction

Attitude control of a rigid body, such as a spacecraft or underwater vehicle, is a problem that has been extensively studied. Various challenges present themselves when attempting attitude control. For instance, the attitude control system must realize the control objectives in the presence of modelling errors, sensor noise, disturbances, and limited on-board actuator authority.

The attitude of a rigid body is described by a direction cosine matrix (or equivalently, by a rotation matrix). Although various authors have studied attitude control using the direction cosine matrix directly [1–5], direction cosine matrix parameterizations are often used in practice. Attitude control methods that use direction cosine matrix parameterizations while simultaneously ensuring asymptotic stability of a desired closed-loop equilibrium point are still of interest to practitioners and theoreticians alike. For instance, unit-length quaternions (referred to as quaternions for simplicity), Gibbs (Rodrigues) parameters, and modified Rodrigues parameters (MRPs) are used within proportional-derivative (PD) type control laws in [6–11]. Adaptive controllers utilizing parameterizations have also been investigated; see [12–15].

Hardware limitations complicate the control of robotic systems such as spacecraft and underwater vehicles. Such systems are equipped with actuators that can only apply a finite amount of torque to the body being controlled. Avoiding actuator saturation while concurrently guaranteeing asymptotic stability of a desired closed-loop equilibrium point is of great practical interest. Modifying PD and similar attitude control laws to disallow the possibility of actuator saturation has been considered in [16–19,15,20]. Often hyperbolic tangent functions are used to prevent saturation [19,20].

A feedback control structure that prohibits actuator saturation and simultaneously guarantees asymptotic stability of a desired closed-loop equilibrium point is developed in this paper. The control is composed of a proportional control term and a dynamic angular velocity control term. The proportional control depends on the attitude parameterization; quaternions, Gibbs parameters,

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and MRPs are considered. The angular velocity control term is realized by a strictly positive real (SPR) system, and hence is a dynamic compensator. Introduction of a SPR system is motivated by the desire to tune the effective angular velocity control over a frequency range in order to, for example, reject sensor noise.

In order to guarantee asymptotic stability of a desired closed-loop equilibrium point and prohibit actuator saturation the theory developed in [21,22] is employed. Specifically, the input to the SPR angular velocity controller is modified by a specific nonlinearity. The novelty of this work lies in the choice and form of the proportional control combined with the SPR angular velocity control with an input nonlinearity that together disallow actuator saturation and at the same time ensures asymptotic stability of a desired closed-loop equilibrium point.

The remainder of this paper is as follows. In Section 2 the kinematics and dynamics of a rigid body are reviewed, as well as SPR systems. The control structure, the saturation nonlinearity, and an additional nonlinearity that is applied to the SPR controller are presented in Section 3.1. Specifically how actuator saturation is prevented is discussed as well. Asymptotic stability of a desired closed-loop equilibrium point is considered in Section 3.2. Based on the work of [23–25] a means to synthesize SPR controllers is presented in Section 3.3. A passive systems interpretation of the control structure is briefly presented in Section 3.4. A numerical example is given in Section 4, and the paper is drawn to a close in Section 5.

2. Preliminaries

The kinematics and dynamics of a rigid body are governed by the Poisson and Euler equations [26]

$$\dot{\mathbf{C}}+\boldsymbol{\omega}^{\times}\mathbf{C}=\mathbf{0}, \tag{1}$$

$$\mathbf{I}\dot{\boldsymbol{\omega}}+\boldsymbol{\omega}^{\times}\mathbf{I}\boldsymbol{\omega}=\boldsymbol{\tau}_c, \tag{2}$$

where $\mathbf{C}\in SO(3)$ is the direction cosine matrix that describes the attitude of the body frame relative to the inertial frame, $SO(3)=\{\mathbf{C}\in\mathbb{R}^{3\times3}|\mathbf{C}^{\mathsf{T}}\mathbf{C}=\mathbf{1},\det\mathbf{C}=+1\}$ is the special orthogonal group of rigid-body rotations, $\boldsymbol{\omega}\in\mathbb{R}^3$ is the angular velocity of the body frame relative to an inertial frame expressed in the body frame, $\mathbf{I}\in\mathbb{R}^{3\times3}$ is the positive definite and symmetric inertia matrix, $\boldsymbol{\tau}_c\in\mathbb{R}^3$ is the control torque, and

$$\boldsymbol{\alpha}^{\times}=\begin{bmatrix}0 & -\alpha_3 & \alpha_2\\ \alpha_3 & 0 & -\alpha_1\\ -\alpha_2 & \alpha_1 & 0\end{bmatrix}$$

is a skew-symmetric matrix satisfying $\boldsymbol{\alpha}^{\times\mathsf{T}}=-\boldsymbol{\alpha}^{\times}$ for any $\boldsymbol{\alpha}\in\mathbb{R}^3$, $\boldsymbol{\alpha}=[\alpha_1\ \alpha_2\ \alpha_3]^{\mathsf{T}}$.

There are many ways to parameterize the direction cosine matrix $\mathbf{C}$ [27]: Euler axis/angle variables, $(\mathbf{a},\phi)$, where $\mathbf{a}\in\mathbb{R}^3$, $\phi\in\mathbb{R}$, and $\mathbf{a}^{\mathsf{T}}\mathbf{a}=1$; quaternions, $\mathbf{q}=[\boldsymbol{\epsilon}^{\mathsf{T}}\ \eta]^{\mathsf{T}}\in\mathbb{S}^3$, where $\boldsymbol{\epsilon}\in\mathbb{R}^3$, $\eta\in\mathbb{R}$, $\boldsymbol{\epsilon}^{\mathsf{T}}\boldsymbol{\epsilon}+\eta^2=1$, and $\mathbb{S}^3=\{\mathbf{q}\in\mathbb{R}^4|\sqrt{\mathbf{q}^{\mathsf{T}}\mathbf{q}}=1\}$; Gibbs parameters, $\mathbf{p}\in\mathbb{R}^3$; and MRPs, $\mathbf{s}\in\mathbb{R}^3$. Quaternions, Gibbs parameters, and MRPs are related to Euler axis/angle variables in the following way [26,27]:

$$\begin{bmatrix}\boldsymbol{\epsilon}\\ \eta\end{bmatrix}=\begin{bmatrix}\mathbf{a}\sin\left(\dfrac{\phi}{2}\right)\\ \cos\left(\dfrac{\phi}{2}\right)\end{bmatrix},\quad \mathbf{p}=\mathbf{a}\tan\left(\frac{\phi}{2}\right),\quad \mathbf{s}=\mathbf{a}\tan\left(\frac{\phi}{4}\right). \tag{3}$$

The direction cosine matrix in terms of these parameters is [26,27]

$$\begin{aligned}\mathbf{C}(\mathbf{a},\phi)&=\cos(\phi)\mathbf{1}+(1-\cos(\phi))\mathbf{a}\mathbf{a}^{\mathsf{T}}-\sin(\phi)\mathbf{a}^{\times},\\ \mathbf{C}(\boldsymbol{\epsilon},\eta)&=(\eta^2-\boldsymbol{\epsilon}^{\mathsf{T}}\boldsymbol{\epsilon})\mathbf{1}+2\boldsymbol{\epsilon}\boldsymbol{\epsilon}^{\mathsf{T}}-2\eta\boldsymbol{\epsilon}^{\times},\end{aligned} \tag{4}$$

$$\begin{aligned}\mathbf{C}(\mathbf{p})&=\mathbf{1}+\frac{2}{(1+\mathbf{p}^{\mathsf{T}}\mathbf{p})}(\mathbf{p}^{\times}\mathbf{p}^{\times}-\mathbf{p}^{\times}),\\ \mathbf{C}(\mathbf{s})&=\mathbf{1}+\frac{1}{(1+\mathbf{s}^{\mathsf{T}}\mathbf{s})^2}(8\mathbf{s}^{\times}\mathbf{s}^{\times}-4(1-\mathbf{s}^{\mathsf{T}}\mathbf{s})\mathbf{s}^{\times}).\end{aligned} \tag{5}$$

The relationship between the angular velocity and the time rate of change of quaternions, Gibbs parameters, and MRPs is [26,27,10,20]

$$\begin{bmatrix}\dot{\boldsymbol{\epsilon}}\\ \dot{\eta}\end{bmatrix}=\frac{1}{2}\begin{bmatrix}\eta\mathbf{1}+\boldsymbol{\epsilon}^{\times}\\ -\boldsymbol{\epsilon}^{\mathsf{T}}\end{bmatrix}\boldsymbol{\omega}, \tag{6}$$

$$\dot{\mathbf{p}}=\tfrac{1}{2}(\mathbf{1}+\mathbf{p}^{\times}+\mathbf{p}\mathbf{p}^{\mathsf{T}})\boldsymbol{\omega}, \tag{7}$$

and

$$\dot{\mathbf{s}}=\frac{1}{2}\left(\left(\frac{1-\mathbf{s}^{\mathsf{T}}\mathbf{s}}{2}\right)\mathbf{1}+\mathbf{s}^{\times}+\mathbf{s}\mathbf{s}^{\mathsf{T}}\right)\boldsymbol{\omega}. \tag{8}$$

Unlike Gibbs parameters and MRPs, which suffer from singularities when $\phi=\pm\pi$ and $\phi=\pm2\pi$, respectively, quaternions are a singularity free attitude representation. However, they do double-cover $SO(3)$ owing to the fact that $\mathbf{q}$ and $-\mathbf{q}$ represent the same physical attitude [3]. Despite the deficiencies of all direction cosine matrix parameterizations they have historically, and continue to be, used in practice for navigation, guidance, and control purposes.

The control framework presented in this paper will use a SPR system as an angular velocity controller. The following lemma provides necessary and sufficient conditions for a linear time-invariant (LTI) system to be SPR.

Lemma 1 (*Kalman-Yakubovich-Popov (KYP) Lemma [28]*). *Consider the LTI system*

$$\dot{\mathbf{x}}_c=\mathbf{A}_c\mathbf{x}_c+\mathbf{B}_c\mathbf{u}_c,\quad \mathbf{y}_c=\mathbf{C}_c\mathbf{x}_c,$$

where $\mathbf{x}_c\in\mathbb{R}^{n_c}$, $\mathbf{u}_c,\mathbf{y}_c\in\mathbb{R}^m$, the matrices $\mathbf{A}_c$, $\mathbf{B}_c$, and $\mathbf{C}_c$ are appropriately dimensioned real matrices that form a minimal state-space realization, and $\mathbf{A}_c$ is Hurwitz. The system is SPR if and only if the exists $\mathbf{P}_c\in\mathbb{R}^{n_c\times n_c}$ and $\mathbf{Q}_c\in\mathbb{R}^{n_c\times n_c}$ where $\mathbf{P}_c=\mathbf{P}_c^{\mathsf{T}}>0$ and $\mathbf{Q}_c=\mathbf{Q}_c^{\mathsf{T}}>0$ such that

$$\mathbf{P}_c\mathbf{A}_c+\mathbf{A}_c^{\mathsf{T}}\mathbf{P}_c=-\mathbf{Q}_c \tag{9a}$$

$$\mathbf{P}_c\mathbf{B}_c=\mathbf{C}_c^{\mathsf{T}}. \tag{9b}$$

The Passivity Theorem states that the negative feedback interconnection of a passive system and an input strictly passive (ISP) system is input–output stable [29].