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Cubature Kalman filtering for relative spacecraft attitude and position estimation

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ABSTRACT

A novel relative spacecraft attitude and position estimation approach based on cubature Kalman filter is derived. The integrated sensor suit comprises the gyro sensors on each spacecraft and a vision-based navigation system on the deputy spacecraft. In the traditional algorithm, an assumption that the chief's body frame coincides with its Local Vertical Local Horizontal (LVLH) frame is made to construct the line-of-sight observations for convenience. To solve this problem, two relative quaternions that map the chief's LVLH frame to the deputy and chief body frames are involved. The general relative equations of motion for eccentric orbits are used to describe the positional dynamics. The implementation equations for the cubature Kalman filter are derived. Simulation results indicate that the proposed filter provides more accurate estimates of relative attitude and position over than the extended Kalman filter.

1. Introduction

Precise relative attitude and position estimation between two spacecrafts are crucial to many space missions, such as spacecraft formation flying, rendezvous and docking, etc. Presently, vision-based navigation (VISNAV) systems, consisting of an optical sensor combined with specific light sources (beacons), are usually used to determine the relative attitude and position within the range of a few hundred meters [1–4]. In order to determine the coupled attitude and position from the line-of-sight (LOS) measurements, a Gaussian Least Squares Differential Correction (GLSDC) algorithm is usually used to provide a deterministic solution. To overcome the shortcomings of iterative computations, Crassidis et al. [5] presented an optimal attitude and position determination approach derived from a generalized predictive filter for nonlinear systems. However, these unitary vision-based estimation methods, which only utilize the static geometrical relations to determine the relative attitude and position, are liable to be affected by the error factors, such as measurement errors, quantization errors, and extracting errors of the beacon locations, etc. Therefore, it is necessary to adopt the state estimation method to design a navigation filter. At present, there are two types of navigation filters that include absolute navigation filter [6–8] and relative navigation filter [1–3,9–10]. The absolute navigation system formulates the dynamics models of the two spacecrafts in the inertial frame and acquires the relative motion parameters by making the differences between two spacecrafts. The relative navigation system estimates the relative position and velocity based on the relative equations of motion established in a rotating Local Vertical Local Horizontal (LVLH) frame. For the close-range relative motion, the relative navigation system is usually adopted due to its sufficient accuracy and better computational efficiency.

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The VISNAV system consists of an optical sensor combined with specific light sources (beacons) to achieve a selective vision. In general, the known beacon locations are defined in the chief body frame, whereas the relative position vector between the chief and deputy spacecrafts is expressed in its LVLH frame. In [1–3], a simplified assumption that chief body frame coincides with its LVLH frame is made to construct the LOS observations for convenience. Unfortunately, this assumption is not valid under all situations, or in some rigorous sense, it is only a special case. One approach to solve this problem is to formulate the relative equations of motion in the chief's body frame [4], and thus the beacon location vectors and relative position vector are described in the same coordinates. The main disadvantages of this approach are that the angular velocity of the chief body frame generally varies rapidly and its measured value is contaminated by the gyro measurement error. They may cause large computation errors in the relative equations of motion. Another approach is to additionally estimate the attitudes of both spacecrafts relative to the LVLH frame [11], and then the assumption that both the chief's body and LVLH frames are the same can be removed.

The extended Kalman filter (EKF) is the most widely used for nonlinear filtering problems so far. However, it works well only in the linear regime in which the linear approximation of the nonlinear dynamic system and observation model is valid. Recently, a cubature Kalman filter (CKF) [12] based on the third-degree spherical-radial cubature rule has been proposed and used in many applications, such as positioning [13], sensor data fusion [14] and attitude estimation [15]. The cubature rule is derivative-free and the number of the scaled cubature points is linearly with the state-vector dimension, which makes the CKF could be applied in high-dimensional nonlinear filtering problems. Compared with the EKF, the CKF has better convergence characteristics and greater accuracy for nonlinear systems [12]. The motivation of this paper is to derive a novel relative spacecraft attitude and position estimation approach based on cubature Kalman filter. In order to maintain the quaternion normalization constraint, an unconstrained three-component vector of generalized Rodrigues parameters (GRPs) is used to propagate and update a nominal quaternion. The performances of the EKF and the CKF with respect to initial condition errors are compared.

This paper is outlined as follows. The cubature Kalman filter is briefly reviewed in Section 2. In Section 3, various reference frames used in this paper are summarized and a review of the relative equations of motion for eccentric orbits is provided. The relative quaternions that map the chief's LVLH frame to the deputy and chief body frames are defined, and the corresponding relative quaternion kinematics equations are given. In Section 4, the gyro measurement model is reviewed and a stringent VISNAV measurement model for the LOS observations is shown. In Section 5, a brief review of the implementation equations for the EKF is shown, and a novel relative spacecraft attitude and position estimation approach based on CKF is derived. In Section 6, simulation results are presented to compare the performances of the EKF and the CKF with respect to initial condition errors. Conclusion remarks are given in Section 7.

2. Cubature Kalman filtering

In this section, a cubature Kalman filter (CKF) that has been developed by Arasaratnam and Haykin is reviewed [12]. The basic idea of the CKF is to use a spherical-radial cubature rule to compute multivariate moment integrals encountered in the nonlinear Bayesian filter. The cubature rule is derivative-free and the number of the scaled cubature points is linearly with the state-vector dimension, which makes the CKF could be applied in high-dimensional nonlinear filtering problems.

Consider the following discrete-time nonlinear dynamical system with additive process and measurement noises:

$$\boldsymbol{x}_k=\boldsymbol{f}(\boldsymbol{x}_{k-1},k)+\boldsymbol{G}_{k-1}\boldsymbol{w}_{k-1} \tag{1}$$

$$\mathbf{z}_k=\boldsymbol{h}(\boldsymbol{x}_k,k)+\boldsymbol{v}_k \tag{2}$$

where $\boldsymbol{x}_k\in\mathbb{R}^n$ is the state vector at time k; $\mathbf{z}_k\in\mathbb{R}^m$ is the measurement vector at time k; $\boldsymbol{f}(\cdot)$ and $\boldsymbol{h}(\cdot)$ are known nonlinear functions; $\boldsymbol{G}_{k-1}$ is discrete-time process noise distribution matrix; $\boldsymbol{w}_{k-1}$ and $\boldsymbol{v}_k$ are independent Gaussian white process noise and measurement noise with covariances $\boldsymbol{Q}_{k-1}$ and $\boldsymbol{R}_k$, respectively.

In the CKF, a third-degree spherical-radial cubature rule is used to approximate an n-dimensional Gaussian weighted integrals as follows:

$$\int_{\mathbb{R}^n}\boldsymbol{f}(\boldsymbol{x})\mathcal{N}(\boldsymbol{x};\boldsymbol{\mu},\Sigma)d\mathbf{x}\approx\frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{f}\left(\boldsymbol{\mu}+\sqrt{\Sigma}\xi_i\right) \tag{3}$$

where $\sqrt{\Sigma}$ is a square-root factor of the covariance Σ which satisfies the relationship $\Sigma=\sqrt{\Sigma}\sqrt{\Sigma}^T$, and the cubature points are given by

$$\xi_i=\begin{cases}\sqrt{n}\boldsymbol{e}_i, & i=1,2,...n\\ -\sqrt{n}\boldsymbol{e}_{i-n} & i=n+1,n+2,...2n\end{cases} \tag{4}$$

where $\boldsymbol{e}_i\in\mathbb{R}^n$ is the i-th elementary column vector. The third-degree cubature rule is exact for Gaussian-weighted integrals whose integrands are written in the form of a linear combination of monomials up to the third degree or any odd-degree.

The entire algorithm is shown as follows:

(a) Initialization:

$$\hat{\boldsymbol{x}}_0=E[\boldsymbol{x}_0],\ \boldsymbol{P}_0=E\left[(\boldsymbol{x}_0-\hat{\boldsymbol{x}}_0)(\boldsymbol{x}_0-\hat{\boldsymbol{x}}_0)^T\right] \tag{5}$$

(b) Time update equations:

1) Evaluate the cubature points $(i=1,2,...2n)$

$$\boldsymbol{P}_{k-1|k-1}=\boldsymbol{S}_{k-1|k-1}\boldsymbol{S}_{k-1|k-1}^T \tag{6}$$

$$\boldsymbol{X}_{k-1|k-1}(i)=\boldsymbol{S}_{k-1|k-1}\xi_i+\hat{\boldsymbol{x}}_{k-1|k-1} \tag{7}$$

2) Evaluate the propagated cubature points $(i=1,2,...2n)$

$$\boldsymbol{X}^*_{k|k-1}(i)=\boldsymbol{f}\left(\boldsymbol{X}_{k-1|k-1}(i)\right) \tag{8}$$

3) Estimate the predicted state and covariance matrix

$$\hat{\boldsymbol{x}}_{k|k-1}=\frac{1}{2n}\sum_{i=1}^{2n}\boldsymbol{X}^*_{k|k-1}(i) \tag{9}$$