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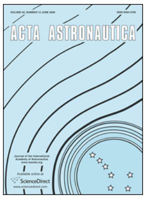

Finite thrust orbital transfers

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ABSTRACT

The finite thrust optimal transfer in the presence of the Earth's shadow and oblate planet perturbations is a problem of strong interest in modern telecommunication satellite design with plasmic propulsion. The Maximum Principle cannot be used in its standard form to deal with the Earth's shadow. In this paper, using a regularization of the Hamiltonian which expands the Maximum Principle application domain, we provide for the first time, the necessary conditions in a very general context for the finite thrust optimal transfer with limited power around an oblate planet. The costate in such problems is generally discontinuous. To obtain fast numerical solutions, the averaging of the Hamiltonian is introduced. Two classes of boundary conditions are analyzed and numerically solved: the minimum time and the minimum fuel at a fixed time. These two problems are the basic tools for designing the orbit raising of a satellite after the launcher injection into its separation orbit. Numerical solutions have been calculated for the more important applications of LEO to GEO/MEO missions and the results have been reported and discussed.

1. Introduction

The finite thrust optimization problem in the presence of the Earth's shadow and perturbations is a significant subject in modern satellite design, because of the current trend to use plasmic propulsion systems for the telecommunication satellite transfer to the Geosynchronous Equatorial Orbit (GEO). Such an approach allows one to reduce the launchers' size and cost for a given telecommunication satellite payload size. The GEO satellites designed with a plasmic propulsion system may have a useful Payload mass up to $\frac{1}{5}$ of the satellite launched mass while those designed with a fully chemical propulsion have a useful Payload mass lower than $\frac{1}{10}$ of the launched mass. This approach is going to reduce significantly the cost per transponder of the GEO telecommunication satellites and will probably affect the policy of the launching systems in the near future, in favor of lighter, restartable and lower cost launchers. Many recent papers like [1,2] deal with such an issue and the European Space Agency has recently implemented a study reported in [3,4] to use plasmic propulsion systems for the in-orbit delivery of the Galileo second generation constellation.

The new approach implies that the satellites must be transferred from a lower, perhaps inclined, orbit to the GEO or MEO operational orbits with the plasmic propulsion. These transfers may take many months, as shown in [5], that describes the long orbit raising of a GEO satellite realized by the author and the Artemis team to recover from a launcher failure.

The mission analysis problem of said transfers consists in determining the trajectory that minimizes the transfer duration or minimizes the propellant consumption for a given fixed transfer duration. The selection of the objective function to be used in the design usually depends on exogenous economic factors like the time to market that

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Nomenclature

0	subscript of the initial orbit
f	subscript of the final orbit
e	eccentricity
a	semi-major axis (km)
g	perigee anomaly from the ascending node
I	inclination of the initial orbit
h	right ascension of the ascending node
u	anomaly from the descending node
ν	anomaly from the perigee
w	orbital rate integrated in time
G	$\sqrt{\mu a(1-e^2)}$, angular momentum (km^2/s) or non-dimensional $G=\sqrt{a(1-e^2)/a_0}$
H	angular momentum normal to the equatorial plane (km^2/s) or non-dimensional $H=H/\sqrt{\mu a_0}$
r	satellite range, dimensional (km) or non-dimensional $r=r/a_0$
p	satellite range rate (km/s)
Y_1	$1/r$
Y_2	w derivative of Y_1
μ	gravitational constant (km^3/s^2)
A	3-vector, acceleration vector (km/s^2)
X	6-vector, non-dimensional orbital state
L	6-vector, non-dimensional orbital costate
$\boldsymbol{X}$	8-vector, non-dimensional orbital state
$\boldsymbol{L}$	8-vector, non-dimensional orbital costate
l	3-vector, firing direction cosines in orbital frame
B	6,3-matrix, maneuver matrix
M	satellite mass (kg)
m	mass flow rate (kg/s)
V_e	gas exhaust velocity (km/s)
f	non-dimensional control force
V	non-dimensional characteristic velocity

the service provider needs for his business: currently we expect that such transfer time may vary from three months to one year depending on the market context.

As shown, for example, in [6] the optimization problem can be treated with a direct or an indirect technique or a hybrid of the two methods. In the direct method the control force vector is discretized in many nodes. This approach for very long transfers brings many thousands of control variables into the numerical algorithm, which results in a slow convergence and inaccurate solutions. This method is often simplified using (see [7]) part of the features of the extremal solutions derived from the Maximum Principle (for a modern introduction see [8,9]).

In the indirect approach, we search for the solution by using the first order necessary conditions of extremality from the Maximum Principle, which results in a Two Boundary Value Problem. No function minima are elaborated, here the difficulty relies in “shooting” the correct boundary conditions with an algorithm, which is not globally convergent in such a highly non-linear context. In this paper we will work exclusively with the indirect method using simplified solutions for initializing the “shooting” algorithm. The analysis of the extremal equations for low thrust transfers is treated in modern papers like [10,7] and classic papers like [11–13]. The usual approach, which is followed in these papers, is to find approximate solutions implementing a control parametrization, then averaging the dynamical equations and finally using the Maximum Principle to optimize the parametrized control. Such approaches allow one, in some cases, to achieve analytical solutions which are very useful in the preliminary analysis and also as initial guess of the shooting methods but may be not very accurate, especially for long transfers.

The Earth's shadow problem, which is very important for long GEO transfers, is not treated in the above-mentioned papers. When the satellite enters the Earth's shadow, the motors are switched off or they are powered by the satellite's battery. This fact implies that a new constraint must be set in the optimization process which requests that no control action is produced in the shadow or when the battery charge level is too low.

The standard indirect method introduced by Edelbaum (see [14]) is still cited today as the reference solver (see [7]). This solver, like also more recent ones (see [15] or [16]), to a certain extent follows our approach. However, in [14] the Earth's shadow constraint is not introduced directly in the Hamiltonian but it is introduced when computing the integration limits of the averaging integral. In this paper we introduce, for the first time, the necessary conditions produced by the Earth's shadow for extremality in a mathematically correct way as the effect of a non-smooth mixed state control constraint. This constraint leads to a discontinuous costate. After that, we introduce the averaging of the Hamiltonian, to produce the so-called “averaged extremal” in which the trajectory is ε-close to the real extremal. It is important to note that when the Earth's shadow is not introduced as a constraint in the Hamiltonian, the final solution is not optimized for the long term Sun's motion and the target function is also more sensitive to the Sun's initial conditions. The quasi-independence of the cost on the Sun's position makes easier the satellite dimensioning task and the mission analysis, so this approach should be applied in all detailed long term orbital transfer calculations.

A recent paper attempting a study of the effect of the Earth's shadow is [17]. In this paper the analysis is limited to a planar, near circular transfer with a fixed shadow. The Hamiltonian defined in [17] does not consider, like [15] or [14], the eclipse mixed state control constraint, which has a negligible effect only if the Sun's motion is negligible during the orbital transfer.

The only paper in the literature to the author's knowledge that recognizes the need to take into account for the mixed state control constraint due to the eclipse is [18]. This paper introduces a regularization method for the Hamiltonian but does not evaluate an explicit form for the optimality conditions in terms of co-state jumps. Another key topic is the averaging. In order to obtain analytic solutions averaging methods are normally introduced with a parametrization of