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Directional passability and quadratic steering logic for pyramid-type single gimbal control moment gyros

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ABSTRACT

Singularity analysis and the steering logic of pyramid-type single gimbal control moment gyros are studied. First, a new concept of directional passability in a specified direction is introduced to investigate the structure of an elliptic singular surface. The differences between passability and directional passability are discussed in detail and are visualized for 0H, 2H, and 4H singular surfaces. Second, quadratic steering logic (QSL), a new steering logic for passing the singular surface, is investigated. The algorithm is based on the quadratic constrained quadratic optimization problem and is reduced to the Newton method by using Gröbner bases. The proposed steering logic is demonstrated through numerical simulations for both constant torque maneuvering examples and attitude control examples.

1. Introduction

A Control Moment Gyro (CMG) system is a momentum exchange actuator used for the attitude control of a spacecraft [1,2]. This system is composed of multiple CMGs installed in the spacecraft, where each CMG contains a wheel spinning at high speed and gimbal rotating structures, and is classified into various types such as Single Gimbal CMG (SGCMG) [3–27], Double Gimbal CMG (DGCMG) [28,29], Variable Speed CMG (VSCMG) [30–33], and Double Gimbal Variable Speed CMG (DGVSCMG) [34] according to the variants of the wheel spinning speed and gimbal rotating structures. An advantage of CMG systems is their efficient torque-producing capability: a relatively small gimbal torque input produces a large torque output on the spacecraft according to the conservation of angular momentum. This makes CMG systems popular for reorienting large space structures and for agile maneuvering of satellites. A disadvantage of CMG systems is the difficulty in designing a CMG steering logic because the gimbal angles change during attitude control. In particular, a CMG system singularity is one of the major obstacles for designing a CMG steering logic. Studies on the singularity problem can be roughly classified into singularity analysis, singularity avoidance, and passability of a singular surface.

Singularity analyses have been carried out by Margulies and Aubrun [3] and by Tokar [4–7] for SGCMG systems. Continuous studies have been conducted by Bedrossian et al. [8], Kurokawa [9,10], Wie [11], Sands et al. [12], and Yamada et al. [13], for SGCMG systems and by Yoon and Tsiotras [32] for VSCMG systems (see [9–11] for a comprehensive survey). In order to analyze the singularity of a CMG system, the angular momentum of the CMG system is expanded to a series of gimbal angles. The CMG system, as well as the angular momentum and gimbal angles, is in a *singular state* if the Jacobian matrix, the first-order coefficient matrix of the series expansion, is deficient. Thus, no control torque can be produced in the sense of the

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first-order approximation along a certain direction, where a unit vector pointing in this direction is called a *singular vector*. The set of all of the angular momentum values in the singular states constitutes the *singular surfaces* in the angular momentum space. Singular surfaces are classified as external singular surfaces and internal singular surfaces. A singular surface is called an *external singular surface* if the magnitude of the angular momentum on the surface is larger than that of the other surfaces along each direction; otherwise, the surface is called an *internal singular surface*. Additional angular momentum for its direction is not possible on an external singular surface by definition; therefore, the external singular surfaces are impassable for any trajectory of the gimbal angular velocity. In contrast, both passable and impassable cases are possible on the internal singular surfaces. Because an internal singular surface exists for any direction inside the external singular surfaces, the trajectory of the angular momentum vector might approach the internal singular surface during the CMG steering. If the angular momentum vector approaches an impassable singular surface, the CMG steering might stick on the surface. Therefore, it is necessary to take into account the internal singular surfaces when designing the CMG steering logic.

Singularity avoidance has been extensively studied [12,14–27,35]. The singularity robust inverse [14–17,35] is simple and suitable for real-time calculations, but the torque error added near a singular surface is an obstacle to highly accurate control. The concept of a constrained workspace [21] is also valid for singularity avoidance, but the restriction of the available angular momentum might degenerate the control performance such as the settling. Feedforward control based on path planning [24–27] is effective in the case of the rest-to-rest maneuver because it is not necessary to perform a back calculation of the gimbal angular velocities from the torque. However, singularity avoidance is difficult in a case where the attitude changes with tracking over the whole trajectory. Singularity avoidance is also difficult in the case of feedback control for the wide-range ground-surface observation by changing the attitude of the spacecraft. Hence, the passability of a singular surface is an important problem for attitude changes with high speed and high accuracy.

The passability of a singular surface is essential in singularity analysis [3–11]. By the definition of a singular state, a singular surface is not passable in the direction of a singular vector in the sense of a first-order approximation. The passability of a singular surface has been evaluated in the sense of a second-order approximation; specifically, a hyperbolic singular surface is passable in the direction of a singular vector by using null motion. An elliptic singular surface is another major obstacle in passing a singular surface. Although the elliptic singular surface is impassable in the direction of a singular vector by using null motion, it might be possible to pass the elliptic singular surface in a certain passing direction by relaxing it from null motion to arbitrary motion. An example of passing along a specific direction is constant torque maneuvering, which is often considered for an agile attitude change. This leads to a discussion on the directional passability in a specific direction. Once the surface is judged to be directionally passable, it is necessary to compute a trajectory for the gimbal angular velocities from a trajectory of the angular momentum; however, a concrete steering logic that generates a precise control torque up to second-order approximation has not been presented in previous studies. This also leads to a discussion of a new steering logic for passing a singular surface. In this study, these problems are addressed for the most typical pyramid-type SGCMG systems. Our first objective is to study the directional passability of a singular surface. The singular surface is *directionally passable* if the angular momentum of CMG systems passes the singular surface along a given direction by using an arbitrary motion. This directional passability will be evaluated in the sense of the second-order approximation. First, the condition for directional passability is obtained for a general direction. The relation for the condition for passability is then discussed. Subsequently, the conditions for directional passability are obtained for the following cases: the direction of a singular vector and the opposite direction of the angular momentum. It will be shown that the singular surface is always directionally passable in the opposite direction of the angular momentum and that the singular surface is passable in any direction if it is passable in the same direction as the angular momentum. This reveals the difficulty of passing a singular surface in the same direction as the angular momentum. The differences between the passability and the directional passability will be visualized in detail for 0H, 2H, and 4H singular surfaces.

The second objective is to investigate a new steering logic, referred to as quadratic steering logic (QSL), for passing a singular surface. QSL is based on a quadratic constrained quadratic optimization problem, i.e., the problem to generate the gimbal angular velocities in response to the control torque up to a second-order approximation such that the magnitude of the gimbal angular velocities is suppressed. It should be noted that the well-known pseudo-inverse steering logic is based on a linear constrained quadratic optimization problem; therefore, QSL is a natural extension of pseudo-inverse steering logic. QSL is computed via numerical optimization. A key idea is to represent the codependence between one free gimbal angle and the other three gimbal angles in the sense of the second-order approximation by using Gröbner bases. Then, the minimization of the quadratic cost of the gimbal angular velocities can be numerically calculated by the Newton method. The effectiveness of the proposed logic will be demonstrated through numerical simulations for two examples. One is for constant torque maneuvering examples for both directionally passable and directionally impassable cases. The other is for attitude control examples, and the logic is compared with singularity avoidance/escape steering logic [17]. It should be noted that the magnitude of the gimbal angular velocities can be sufficiently suppressed for a sufficiently small control time interval in directionally passable cases; however, the magnitude of the gimbal angular velocities cannot be always suppressed in directionally impassable cases. This requires accounting for the saturation of the gimbal angular velocities for implementation; therefore, the modified logic is included in attitude control examples.