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Dynamical evolution of natural formations in libration point orbits in a multi-body regime

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ABSTRACT

This investigation explores the natural dynamics in a multi-body regime for formation flying applications. Natural regions that are suitable to maintain multiple spacecraft in a loose formation are determined in an inertial coordinate frame. Locating a formation of spacecraft in these zones leads to a smaller variation in the mutual distance between the spacecraft and the pointing direction of the formation. These suitable regions approximate quadric surfaces along a reference trajectory and the relationship between the dynamical evolution of the quadric surfaces and the eigenstructure of the reference trajectory is examined.

1. Introduction

Spacecraft formations offer many potential applications in the future of space exploration, including the search for habitable terrestrial exoplanets, the identification of black holes, and many others. During the last decade, due to the detection of a large number of extrasolar planets, new studies on formation flying in multi-body regimes have emerged to support space astronomy. For example, the original New Worlds Observer (NWO) design concept employs a telescope and an external occulter for the detection and characterization of Earth-like planets [1]. During the observation of the star, the distance between the two spacecraft and the pointing direction of the formation toward the star line-of-sight are maintained constant. The occulter suppresses the starlight by many orders of magnitude, to enable the observation of habitable terrestrial planets and the detection of life signs.

The L_2 Sun–Earth libration point region has been a popular destination for satellite imaging formations. Barden and Howell investigate the natural behavior on the center manifold near the L_2 Sun–Earth libration point and compute some natural six-spacecraft formations, which demonstrate that quasi-periodic trajectories could be useful for formation flying [2]. Later, Marchand and Howell extend this study and use some control strategies, continuous and discrete, to maintain non-natural formations near the libration points [3]. Space-based observatory and interferometry missions, such as the Terrestrial Planet Finder, have been the motivation for the analysis of many control strategies. Gómez et al. investigate discrete control methods to maintain a formation of spacecraft [4]. Howell and Marchand consider linear optimal control, as applied to nonlinear time-varying systems, as well as nonlinear control techniques, including input and output feedback linearization [5]. Hsiao and Scheeres implement position and velocity feedback control laws to force a formation of spacecraft to possess a rotational motion

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relative to a nominal trajectory [6,7]. Recently, Gómez et al. derive regions around a halo orbit with zero relative velocity and zero relative radial acceleration that ideally maintain the mutual distances between spacecraft [8]. Most recently, Héritier and Howell then investigate the natural dynamics in the collinear libration point region for the control of formations of spacecraft [9,10]. This current analysis details and expands on the initial work of Héritier and Howell. A more general understanding of these natural regions suitable for formation flight is examined in the inertial frame for the placement of a small formation of spacecraft along various reference trajectories.

Controlling multiple spacecraft in a multi-body environment is challenging and a good understanding of the natural dynamics in this regime is essential. Hence, this investigation explores the dynamical environment near the L_2 Sun–Earth libration point to aid in the control of a formation of spacecraft, in terms of the relative distance between vehicles. Regions with low natural drift that are suitable to maintain multiple spacecraft in a loose formation are determined in an inertial frame. Locating a formation of spacecraft in these zones leads to a smaller variation in the mutual distance between the spacecraft and the pointing direction of the formation. The characteristics of these zones are then investigated in detail. These suitable regions are derived analytically using variational equations relative to the reference path. They represent quadric surfaces and the relationship between the dynamical evolution of the quadric surfaces and the eigenstructure of the reference trajectory is examined.

2. Dynamical model

For the analysis of spacecraft formations in a multi-body regime, the Circular Restricted Three-Body Problem (CR3BP) is selected to describe the motion of the spacecraft. The Sun and the Earth are selected as the two gravitational bodies for this investigation. The analysis of the relative motion between vehicles is expressed in an inertial coordinate frame. The reference trajectories considered in this analysis are first computed in a rotating frame relative to the primary bodies and then transposed in the inertial frame. A chief spacecraft is assumed to move along a given reference trajectory.

2.1. Relative motion described in the inertial frame

The equations of motion are described in an inertial coordinate frame where the spacecraft is located relative to the barycenter of the Sun and the Earth [11]. Let $\overline{X}$ define a general vector in the inertial frame describing the motion of the spacecraft, i.e.,

$$\overline{X}(t) = [x \;\; y \;\; z \;\; \dot{x} \;\; \dot{y} \;\; \dot{z}]^T \tag{1}$$

where superscript ‘T’ implies transpose. The dynamical system is defined as

$$\dot{\overline{X}}(t) = \overline{f}(\overline{X}, t) \tag{2}$$

The scalar nonlinear equations of motion in their non-dimensional forms expressed in the inertial frame are then written as

$$\ddot{x} = (1-\mu)(x_S - x)/r_{13}^3 + \mu(x_E - x)/r_{23}^3 \tag{3}$$

$$\ddot{y} = (1-\mu)(y_S - y)/r_{13}^3 + \mu(y_E - y)/r_{23}^3 \tag{4}$$

$$\ddot{z} = (1-\mu)(z_S - z)/r_{13}^3 + \mu(z_E - z)/r_{23}^3 \tag{5}$$

with

$$r_{13} = \sqrt{(x-x_S)^2 + (y-y_S)^2 + (z-z_S)^2} \tag{6}$$

$$r_{23} = \sqrt{(x-x_E)^2 + (y-y_E)^2 + (z-z_E)^2} \tag{7}$$

and where μ is the mass parameter associated with the Sun–Earth system. The positions of the Sun and the Earth are defined as (x_S, y_S, z_S) and (x_E, y_E, z_E), respectively. They are expressed as

$$x_S = -\mu \cos(t-t_0), \quad x_E = (1-\mu)\cos(t-t_0) \tag{8}$$

$$y_S = -\mu \sin(t-t_0), \quad y_E = (1-\mu)\sin(t-t_0) \tag{9}$$

$$z_S = 0, \quad z_E = 0 \tag{10}$$

where the initial time is assumed to be zero, i.e., $t_0 = 0$. The right-hand side of Eqs. (3)–(5) can be expanded to first-order in μ as

$$\ddot{x} = -x/r^3 + \mu F(x,y,z,t,t_0) + O(\mu^2) \tag{11}$$

$$\ddot{y} = -y/r^3 + \mu G(x,y,z,t,t_0) + O(\mu^2) \tag{12}$$

$$\ddot{z} = -z/r^3 + \mu H(x,y,z,t,t_0) + O(\mu^2) \tag{13}$$

where $r = \sqrt{x^2+y^2+z^2}$, and

$$F(x,y,z,t,t_0) = \frac{x-\cos(t-t_0)}{r^3} + \frac{3x(x\cos(t-t_0) + y\sin(t-t_0))}{r^5} + \frac{\cos(t-t_0) - x}{((x-\cos(t-t_0))^2 + (y-\sin(t-t_0))^2 + z^2)^{3/2}} \tag{14}$$

$$G(x,y,z,t,t_0) = \frac{y-\sin(t-t_0)}{r^3} + \frac{3y(x\cos(t-t_0) + y\sin(t-t_0))}{r^5} + \frac{\sin(t-t_0) - y}{((x-\cos(t-t_0))^2 + (y-\sin(t-t_0))^2 + z^2)^{3/2}} \tag{15}$$

$$H(x,y,z,t,t_0) = \frac{z}{r^3} + \frac{-z}{((x-\cos(t-t_0))^2 + (y-\sin(t-t_0))^2 + z^2)^{3/2}} \tag{16}$$

The design process relies on variations relative to a reference trajectory. Given a solution to the nonlinear differential equations, linear variational equations of motion are derived from a first-order Taylor expansion defined as

$$\delta\dot{\overline{X}}(t) \approx \frac{\partial \overline{f}}{\partial \overline{X}}_{|\overline{X}-\overline{X}_{ref}} \delta\overline{X}(t) + \frac{\partial \overline{f}}{\partial t}_{|t-t_{ref}} \delta t + O\left(\delta\overline{X}^2, \delta t^2\right) \tag{17}$$