Acta Astronautica 98 (2014) 120–127

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro

Thrust-vector control of a three-axis stabilized upper-stage rocket with fuel slosh dynamics

Jaime Rubio Hervas [1], Mahmut Reyhanoglu *

Physical Sciences Department, Embry-Riddle Aeronautical University, Daytona Beach, FL 32114, USA

ARTICLE INFO

Article history:
Received 14 June 2013
Received in revised form
30 October 2013
Accepted 26 January 2014
Available online 1 February 2014

Keywords:
Thrust-vector control
Propellant slosh
Lyapunov methods

ABSTRACT

This paper studies the thrust vector control problem for an upper-stage rocket with fuel slosh dynamics. The dynamics of a three-axis stabilized spacecraft with a single partially-filled fuel tank are formulated and the sloshing propellant is modeled as a multi-mass–spring system, where the oscillation frequencies of the mass–spring elements represent the prominent sloshing modes. The equations of motion are expressed in terms of the three-dimensional spacecraft translational velocity vector, the attitude, the angular velocity, and the internal coordinates representing the slosh modes. A Lyapunov-based nonlinear feedback control law is proposed to control the translational velocity vector and the attitude of the spacecraft, while attenuating the sloshing modes characterizing the internal dynamics. A simulation example is included to illustrate the effectiveness of the control law.

1. Introduction

The dynamics of space vehicles with liquid propellant tanks are adversely affected by sloshing that is induced by propellant tank motions resulting from guidance and control system commands or from changes in vehicle acceleration. To mitigate the destabilizing effects of the interaction of the slosh dynamics with the rigid body dynamics of the spacecraft, a variety of passive techniques have been employed, including the use of baffles, bladders, and partitions [1]. These passive techniques increase energy dissipation and limit the movement of liquid fuel. However, the passive techniques do not completely succeed in cancelling the sloshing effects. Moreover, these suppression methods add mass and structural complexity to the vehicle. Therefore, it is desirable to control the sloshing using active methods.

It is known that pendulum and mass–spring models can approximate complicated fluid and structural dynamics; such models have formed the basis for many studies on the dynamics and control of space vehicles with fuel slosh [2–10].

Several linearization-based control design approaches have been developed for space vehicles to suppress the fuel slosh dynamics [11–17]. In most of these approaches, suppression of the slosh dynamics inevitably leads to the excitation of the transverse vehicle motion through coupling effects; this is a major drawback of linearization-based control laws.

The previous work by the authors considered the nonlinear modeling and control problems for planar maneuvering of space vehicles with fuel slosh dynamics [6–10]. Both multi-mass–spring models [6–9] and multi-pendulum models [10] were used for the characterization of the most prominent sloshing modes. This paper extends these works to the three-dimensional maneuvers of a spacecraft with fuel slosh dynamics.

In this paper, a three-axis stabilized upper-stage rocket with a partially filled fuel tank is considered, and prominent slosh modes are included in the dynamic model using mass–spring analogy. The control inputs are the two

* Corresponding author. Tel.: +1 386 226 7753; fax: +1 386 226 6621.
E-mail address: reyhanom@erau.edu (M. Reyhanoglu).
[1] Present address: School of Mechanical and Aerospace Engineering at Nanyang Technological University, Singapore.

0094-5765/$ - see front matter © 2014 IAA. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.actaastro.2014.01.022

Nomenclature

$\mathbf{v}=[v_x, v_y, v_z]^T$ spacecraft translational velocity vector, m/s
$\boldsymbol{\omega}=[\omega_x, \omega_y, \omega_z]^T$ spacecraft angular velocity vector, rad/s
$\boldsymbol{\theta}=[\theta_x, \theta_y, \theta_z]^T$ Euler angles, rad
$\mathbf{s}$ vector of internal coordinates
$L=T-U$ Lagrangian (kinetic energy minus elastic potential energy), J
R Rayleigh dissipation function, N m/s
$\boldsymbol{\tau}_t$ vector of generalized control forces, N
$\boldsymbol{\tau}_r$ vector of generalized control torques, N m
$\hat{\mathbf{p}}$ cross-product operation $\mathbf{p}\ \times$
xyz spacecraft-fixed principal axes
F thrust, N
(δ_1, δ_2) gimbal deflection angles about the y and z axes, rad
$\mathbf{M}=[M_x, M_y, M_z]^T$ input torque vector, N m
N number of slosh modes
m_0 mass of still liquid propellant, kg
I_0 moment of inertia of still liquid propellant, kg m^2
m_i mass corresponding to ith slosh mode, kg
ξ_i relative position of mass m_i along the y-axis, m
η_i relative position of mass m_i along the z-axis, m
h_0 location of mass m_0 along the x-axis, m
h_i location of mass m_i along the x-axis, m
k_i spring constant corresponding to ith slosh mode, N/m
c_i damping constant corresponding to ith slosh mode, N s/m
$\mathbf{J}$ spacecraft principal moment of inertia tensor, kg m^2
b spacecraft center of mass (CM) location along the x-axis, m
d distance from the gimbal pivot to the spacecraft CM, m
p gimbal pivot location along the x-axis, m
m_f total fuel mass, kg
$\mathbf{r}=[x, y, z]^T$ inertial position of CM of the undisturbed fuel, m
$\mathbf{r}_c$ inertial position of the spacecraft CM, m
$\mathbf{v}_c$ inertial velocity of the spacecraft CM, m/s
$\mathbf{r}_0, \mathbf{r}_i$ inertial positions of fuel masses m_0 and m_i, m
$\mathbf{v}_0, \mathbf{v}_i$ inertial velocities of fuel masses m_0 and m_i, m/s
$\mathbf{a}=[a_x, a_y, a_z]^T$ acceleration of the CM of the fuel, m/s^2
$\mathbf{I}$ 3×3 identity matrix
v_{x_0} initial axial velocity of the spacecraft, m/s
t_b fuel burn time, m
$(u_1, u_2, u_3, u_4, u_5)$ transformed control inputs
Ω_i undamped natural frequency of the ith slosh mode, rad/s
ζ_i damping ratio of the ith slosh mode
r_i, l_i positive control parameters
$\mathrm{Bo}=\rho a\mathcal{R}^2/\sigma$ Bond number
ρ liquid propellant's density, kg/m^3
σ surface tension, kg/s^2
$\mathcal{R}$ characteristic dimension of the propellant tank, m

gimbal deflection angles of a main engine and three independent torques, generated by either gas jet pairs or reaction wheels, about the center of mass of the spacecraft. It is assumed that the rocket acceleration due to the main engine thrust is large enough so that surface tension forces do not significantly affect the propellant motion during main engine burns. The control objective is to control the translational velocity vector and the attitude of the spacecraft, while attenuating the sloshing modes characterizing the internal dynamics. The results are applied to the AVUM upper stage–the fourth stage of the European launcher Vega [18]. The main contributions of this paper are (i) the development of full nonlinear mathematical models for 3D maneuvering of the spacecraft with fuel slosh dynamics and (ii) the design of a Lyapunov-based nonlinear feedback control law. Simulation results are included to illustrate the effectiveness of the controller.

2. Model formulation

This section formulates the dynamics of a three-axis stabilized spacecraft with a single propellant tank including the prominent fuel slosh modes. The spacecraft is represented as a rigid body (base body) and the sloshing fuel masses as internal bodies. The equations of motion are expressed in terms of the three-dimensional spacecraft translational velocity vector, the attitude, the angular velocity, and the internal (shape) coordinates representing the slosh modes.

2.1. Multibody vehicle dynamics

Following the development in [8,19], let $\mathbf{v}\in\mathbb{R}^3$, $\boldsymbol{\omega}\in\mathbb{R}^3$, and $\mathbf{s}\in\mathbb{R}^q$ denote the base body translational velocity vector, the base body angular velocity vector, and the vector of internal coordinates, respectively. In these variables, the Lagrangian has the form $L=L(\mathbf{v},\boldsymbol{\omega},\mathbf{s},\dot{\mathbf{s}})$, which is $SE(3)$-invariant in the sense that it does not depend on the base body position and attitude. The internal dissipative forces are assumed to be derivable from a Rayleigh dissipation function R. Then, the equations of motion of the spacecraft with internal dynamics are shown to be given by

$$\frac{d}{dt}\frac{\partial L}{\partial \mathbf{v}}+\hat{\boldsymbol{\omega}}\frac{\partial L}{\partial \mathbf{v}}=\boldsymbol{\tau}_t, \quad (1)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \boldsymbol{\omega}}+\hat{\boldsymbol{\omega}}\frac{\partial L}{\partial \boldsymbol{\omega}}+\hat{\mathbf{v}}\frac{\partial L}{\partial \mathbf{v}}=\boldsymbol{\tau}_r, \quad (2)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{s}}}-\frac{\partial L}{\partial \mathbf{s}}+\frac{\partial R}{\partial \dot{\mathbf{s}}}=0, \quad (3)$$