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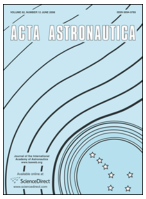

Launch ascent guidance by discrete multi-model predictive control

Alexandre Vachon [a,*], André Desbiens [a], Eric Gagnon [b], Caroline Bérard [c]

[a] Université Laval, Pavillon Adrien-Pouliot, 1065 avenue de la Médecine, Québec, Canada G1V 0A6
[b] Defence R&D Canada – Valcartier, 2459 boulevard Pie-XI Nord, Québec, Canada G3J 1X5
[c] Institut Supérieur de l'Aéronautique et de l'Espace, 10 avenue Édouard-Belin BP 54032, 31055 Toulouse Cedex 4, France

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ABSTRACT

This paper studies the application of discrete multi-model predictive control as a trajectory tracking guidance law for a space launcher. Two different algorithms are developed, each one based on a different representation of launcher translation dynamics. These representations are based on an interpolation of the linear approximation of nonlinear pseudo-five degrees of freedom equations of translation around an elliptical Earth. The interpolation gives a linear-time-varying representation and a linear-fractional representation. They are used as the predictive model of multi-model predictive controllers. The controlled variables are the orbital parameters, and constraints on a terminal region for the minimal accepted precision are also included. Use of orbital parameters as the controlled variables allows for a partial definition of the trajectory. Constraints can also be included in multi-model predictive control to reduce the number of unknowns of the problem by defining input shaping constraints. The guidance algorithms are tested in nominal conditions and off-nominal conditions with uncertainties on the thrust. The results are compared to those of a similar formulation with a nonlinear model predictive controller and to a guidance method based on the resolution of a simplified version of the two-point boundary value problem. In nominal conditions, the model predictive controllers are more precise and produce a more optimal trajectory but are longer to compute than the two-point boundary solution. Moreover, in presence of uncertainties, developed algorithms exhibit poor robustness properties. The multi-model predictive control algorithms do not reach the desired orbit while the nonlinear model predictive control algorithm still converges but produces larger maneuvers than the other method.

1. Introduction

It is well known that there are two approaches to guide a vehicle toward its final destination [1]: predictor/corrector methods and path reference methods. Launch ascent guidance of a space launcher is no different than any other vehicle, and both approaches have been applied to it in the past [2]. The predictor/corrector approaches consist in the generation of a new trajectory and the corresponding steering commands at each iteration where the current state is the initial state of a two-point boundary value problem. Hence, complex optimization algorithms are required to solve the problem, or a hypothesis can be formulated to simplify the problem. For the launch ascent trajectory, two hypotheses on the Earth gravity approximation, proportional to the radius [3] or uniform [4], give an analytical solution of the costate system and eliminate the need for

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[*] Corresponding author. Tel.: +1 418 656 2131x5652.
E-mail addresses: alexandre.vachon.2@ulaval.ca (A. Vachon), Andre.Desbiens@gel.ulaval.ca (A. Desbiens), Eric.Gagnon@drdc-rddc.gc.ca (E. Gagnon), Caroline.Berard@isae.fr (C. Bérard).

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complex algorithms. Both of these hypotheses are well studied and can be considered mature techniques suitable for many launch applications [4]. The path reference methods compute the steering commands needed to follow a predefined trajectory. As presented by Shrivastava et al. [2], many linear control methods with time scheduled values have been studied for launch ascent guidance. Among them, Q-guidance is the most important one [5]. Recent developments in nonlinear control theory boost the path reference approaches as linearization of the nonlinear equations of motion is not required. Nonlinear model predictive control (NMPC) [6], neural networks [7] and sliding mode control [8] are examples of the application of nonlinear control methods to the guidance of a space launcher.

Even if, under nominal conditions, the trajectory produced by the path reference methods is closer to the optimal trajectory than the trajectory obtained by the predictor/corrector methods [2], launch ascent guidance is traditionally achieved by predictor/corrector methods [9]. The main reason for this choice is the robustness of the predictor/corrector methods under non-nominal conditions; however, in the exo-atmospheric portion, space launchers are expected to follow their nominal trajectory quite closely as they evolve in an environment that is practically perturbation-free. Therefore, path reference must be considered for this portion of the launch, mainly for vehicles with well-defined missions and reliable components [2].

This paper focuses on MPC-based path reference approaches. Through integral resolution, continuous time predictive control of Lu [6] restricts tracking to the in-plane ascent trajectory around a spherical Earth. The proposed guidance laws are discrete versions of predictive control. In opposition to continuous predictive control, discrete formulation has a finite number of unknowns and does not require the model to have a special form to obtain the solution. Therefore, the in-plane and out-of-plane ascent problem of a motion around an elliptical Earth can be solved. The discrete model is an Euler approximation of the pseudo-5 degrees of freedom (pseudo-5DoF) equations of motion. The prediction of a NMPC is made directly with these equations of motion while the varying linear representations are the bases of multi-model predictive control (MMPC) algorithms. Vachon et al. [10] apply discrete NMPC to space launcher exo-atmospheric guidance. The varying linear representations are time interpolation (linear-time-varying representation (LTVR) and linear-fractional representation (LFR)) of a set of linear models of the launcher translation dynamics [11].

MMPC algorithms based on LTVR and LFR are well-known approaches used to handle multiple operation regimes [12–15]. That being said, they are mostly implemented as linear MPC where the predictive model varies at each iteration [12] or where multiple linear MPC algorithms are simultaneously solved and a second algorithm weighs the results based on the current operating regime [14]. These formulations are valid for systems operating in multiple operating regimes but where the variations are slow and hence the prediction around a single operating regime is valid. When the operating regime varies inside the prediction horizon, the algorithms are based on minmax optimization to obtain the varying model dynamically [13,15]. This is a good approach when the sequence of the operating regimes is not known *a priori*. In a launcher guidance law, the prediction must be made over multiple operating regimes that are known *a priori*. Hence, identifying the varying linear model using minmax optimization at each time step is not necessary, and pre-identified models can be used. The minmax algorithm is therefore converted into a single minimization.

Section 2 gives an overview of the representations of the launcher translation dynamics obtained by Vachon et al. [11]. Section 3 introduces the NMPC and the two MMPC formulations. The resulting formulations are then implemented and tested in simulations. Section 4 compares the results with a specific predictor/corrector method [3].

2. Equations of motion

2.1. Pseudo-five degrees of freedom equations

As stated by Zipfel [16], a guidance law developed for a pseudo-5DoF can be implemented in a full six-degree simulator with no modifications. Pseudo-5DoF equations are composed of the three degrees of freedom equations of translation and the pseudo-two degrees of freedom approximating the rotational dynamics. The polar coordinates version of the translation equations is used and the controlled rotational dynamics are approximated by two first-order transfer functions (one for the in-plane motion and another for the out-of-plane motion). A time constant of 1 s for both transfer functions is coherent with the requirements of an exo-atmospheric control function [17]. Combining these two sets of equations gives the complete nonlinear pseudo-5DoF equations:

$$\dot{m} = \Delta m \tag{1a}$$

$$\dot{r} = v\ \sin(\gamma) \tag{1b}$$

$$\begin{aligned}\dot{v} = {} & \frac{T\ \cos\ \vartheta\ \cos\ \varphi}{m} - g_r\ \sin\ \gamma + g_\delta\ \cos\ \chi\ \cos\ \gamma \\ & -\omega_e^2 r\ \cos\ \delta(\sin\ \delta\ \cos\ \chi\ \cos\ \gamma - \cos\ \delta\ \sin\ \gamma)\end{aligned} \tag{1c}$$

$$\dot{\delta} = \frac{v\ \cos\ \gamma\ \cos\ \chi}{r} \tag{1d}$$

$$\dot{\lambda} = \frac{v\ \cos\ \gamma\ \sin\ \chi}{r\ \cos\ \delta} \tag{1e}$$

$$\begin{aligned}\dot{\chi} = {} & \frac{T\ \cos\ \vartheta\ \sin\ \varphi}{mv\ \cos\ \gamma} + \frac{v}{r}\ \sin\ \chi\ \cos\ \gamma\ \tan\ \delta \\ & - \frac{g_\delta\ \sin\ \chi}{v\ \cos\ \gamma} + \frac{\omega_e^2}{v\ \cos\ \gamma} r\ \cos\ \delta\ \sin\ \delta\ \sin\ \chi \\ & - \frac{2\omega_e}{\cos\ \gamma}(\sin\ \gamma\ \cos\ \delta\ \cos\ \chi - \cos\ \gamma\ \sin\ \delta)\end{aligned} \tag{1f}$$

$$\begin{aligned}\dot{\gamma} = {} & \frac{-T\ \sin\ \vartheta}{mv} - \frac{g_r}{v}\ \cos\ \gamma - \frac{g_\delta}{v}\ \cos\ \chi\ \sin\ \gamma \\ & + \frac{v}{r}\ \cos\ \gamma + 2\omega_e\ \sin\ \chi\ \cos\ \delta \\ & + \frac{\omega_e^2 r\ \cos\ \delta}{v}(\cos\ \gamma\ \cos\ \delta + \sin\ \delta\ \cos\ \chi\ \sin\ \gamma)\end{aligned} \tag{1g}$$