Acta Astronautica 95 (2014) 111–123

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro

Size-dependent dynamic pull-in instability of vibrating electrically actuated microbeams based on the strain gradient elasticity theory

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ARTICLE INFO

Article history:
Received 18 February 2013
Received in revised form
17 September 2013
Accepted 30 October 2013
Available online 8 November 2013

Keywords:
Strain gradient elasticity theory
Dynamic pull-in instability
Size-dependent
Parameter Expansion Method
Frequency–amplitude relationship
Microbeam vibration

ABSTRACT

This paper presents the impact of vibrational amplitude on the dynamic pull-in instability and fundamental frequency of actuated microbeams by introducing the second order frequency–amplitude relationship. The nonlinear governing equation of microbeam predeformed by an electric force including the fringing field effect, based on the strain gradient elasticity theory is considered. The predicted results of the strain gradient elasticity theory are compared with the outcomes that arise from the classical and modified couple stress theory. The influences of basic nondimensional parameters on the pull-in instability as well as the natural frequency are investigated by a powerful asymptotic approach namely the Parameter Expansion Method (PEM). It is demonstrated that two terms in series expansions are sufficient to produce an acceptable solution of the microstructure. The phase portrait of the microstructure shows that by increasing the actuation voltage parameter, the stable center point loses its stability and coalesces with unstable saddle node.

1. Introduction

Recent advances in microelectronics technology and production processes have led to the rapid acceleration in microelectromechanical systems (MEMS) applications. MEMS devices that integrate electrical and mechanical elements, because of their small size and weight, are widely used in inkjet printers [1], airbag deployment systems [2], automobile stability control systems [3], Digital Light Processing (DLP) technology [4], micropumps [5], optical switches [5], microtweezers [6] and micromirrors [7]. Recently several numerical and experimental studies have been conducted on the pull-in instability and dynamic behavior of MEMS devices [8–13]. In the dynamic analysis of microsystems, electrostatic forces cause the relationship between the input excitation and the output response to be nonlinear. However, the amplitude dependence of fundamental frequency and pull-in instability has not been developed, till present.

Pull-in and snap-through instabilities in transient deformations of microelectromechanical systems have been presented by Das and Batra [14]. They revealed that with a decrease in the rate of the applied potential difference, the pull-in and the snap-through parameters approach those for a static problem. Fu and Zhang [15] discussed the surface energies on the static and dynamic responses, pull-in voltage and pull-in time of electrically actuated nano-beams by applying the Gurtin and Murdoch's theory of surface. They also included the effect of the geometrical nonlinearity in their study. Wang et al. [16] established the mathematical model to estimate the pull-in parameters and the fundamental frequency in a prestressed multi-layer microbeam with nonlinearities such as electrostatic loads and large deformation. Jia et al. [17] investigated

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http://dx.doi.org/10.1016/j.actaastro.2013.10.020

the pull-in instability of microswitches under the combined electrostatic and intermolecular forces and axial residual stress. They obtained pull-in voltage and pull-in deflection for microswitches with four different boundary conditions using the differential quadrature method (DQM). Sedighi and Shirazi [18] presented a new asymptotic procedure to predict the nonlinear vibrational behavior of classical microbeams predeformed by an electric field. They investigated the influences of basic parameters on pull-in instability and natural frequency of microstructures. Rahaeifard et al. [19] studied the dynamic behavior of microcantilevers under sudden applied DC voltage based on the modified couple stress theory. They employed the multiple scales method (MSM) and the finite difference method to predict the dynamic behavior of the microbeams. Rajabi and Ramezani [20] developed a microscale nonlinear beam model based on strain gradient elasticity. Vyasarayani et al. [21] developed a mathematical model of an electrostatic MEMS beam undergoing impact with a stationary electrode subsequent to pull-in. They modeled the contact between the beam and the substrate using a nonlinear foundation of springs and dampers. Towfighian et al. [22] presented the closed-loop dynamics of a chaotic electrostatic microbeam actuator and plotted the bifurcation diagrams which were obtained by sweeping the AC voltage amplitudes and frequency. Chouvion et al. [23] reviewed several approaches for calculating semi-infinite support loss in microelectromechanical system resonators undergoing in-plane vibration. The symmetry breaking, the snap-through instability and the pull-in instability of bi-stable arch-shaped MEMS under static and dynamic electric loads were investigated by Das and Batra [24]. They constructed the phase diagram between a critical load parameter and the arch height to delineate different regions of instabilities.

Nowadays, substantial progresses had been made in analytical solutions for nonlinear equations without small parameters [25–27]. Several approaches have been employed to solve the governing nonlinear differential equations to study the nonlinear vibrations such as the Energy Balance Method (EBM) [28], the Variational Iteration Method (VIM) [29], the Lindstedt–Poincaré method [30], the Laplace iteration method [31], the Max–Min approach (MMA) [32], the Homotopy Analysis Method (HAM) [33], the Parameter Expansion Method (PEM) [34,35], the Homotopy Perturbation Method (HPM) [36], the Hamiltonian Approach (HA) [37], the Iteraton Perturbation Method and the Homotopy Perturbation Method with Auxiliary Term [38].

The aim of the present article is to investigate the impact of vibrational amplitude on pull-in instability and natural frequency of actuated microbeams by introducing the second order frequency–amplitude relationship. The nonlinear governing equation of microbeam with electrostatic force including fringing fields, based on the strain gradient elasticity is investigated. To this end, analytical expressions for vibrational response of actuated microbeam are presented. The proposed analytical approach demonstrates that two terms in series expansions are sufficient to obtain a highly accurate solution of microbeam vibration. Finally, the influences of amplitude of vibration and significant parameters on the pull-in instability behavior are studied.

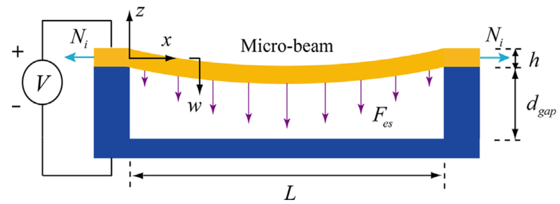

Fig. 1. The configuration of an electrostatically actuated microbeam.

2. Mathematical formulation

Consider a clamped–clamped microbeam suspended above a rigid substrate and under electrostatically actuated voltage as shown in Fig. 1. The microstructure has length L, thickness h, width b, density ρ, moment of inertia I and a modulus of elasticity E. The air initial gap is d_{gap} and an attractive electrostatic force which originates from actuation voltage V causes the microbeam to deflect. Von Karman nonlinearity and small strains of the narrow microbeam are taken into account. The governing equation of motion of microactuated beam based on the strain gradient elasticity theory is expressed as follows:

$$\rho A_0 w_{tt} + \left[EI + \frac{1}{15}\mu A_0(30l_0^2 + 8l_1^2 + 15l_2^2)\right] w_{xxxx} - \frac{2}{5}\mu I(5l_0^2 + 2l_1^2)w^{(\mathrm{vi})} - \left(N_i + \frac{Ebh}{2L}\int_0^L w_x^2\,dx\right) w_{xx} - F_{es} = 0 \tag{1}$$

where μ, A_0, and N_i represent the shear modulus, the cross section area of microbeam and the initial axial load and l_0, l_1 and l_2 denote the additional independent material length scale parameters associated with the dilatation gradients, deviatoric stretch gradients and symmetric rotation gradients, respectively [20]. A uniform electric field cannot drop abruptly to zero at an edge. In actual situation, the fringing field always exists, and a more realistic situation including “fringing-field” modification field must be taken into consideration. The electrostatic force per unit length can be written as [39,40]

$$F_{es} = \frac{1}{2}\frac{b\varepsilon V^2}{(d_{gap} - w)^2}(1 + f_f) \tag{2}$$

where $\varepsilon = 8.854187817620 \times 10^{-12}$ F/m is the vacuum permittivity and the parameter f_f represents the first order fringing-field correction and may be expressed as

$$f_f = \beta\frac{d_{gap} - w}{b} \tag{3}$$

In the aforementioned equation, β is set to 0.65 for a clamped–clamped microbeam [40]. By introducing the following nondimensional variables:

$$\tau = \sqrt{\frac{EI}{\rho b h L^4}}t,\quad W = \frac{w}{d_{gap}},\quad \xi = \frac{x}{L},\quad \alpha = 6\left(\frac{d_{gap}}{h}\right)^2,$$
$$\lambda^2 = \frac{24\varepsilon L^4 V^2}{Eh^3 d_{gap}^3},\quad f_i = \frac{N_i L^2}{EI},\quad \gamma = \frac{d_{gap}}{b},\quad \mu_s = \frac{12\mu}{E(h/l_2)^2} \tag{4}$$

the nondimensional nonlinear governing equation of motion based on the strain gradient elasticity theory can