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Small gain stability theory for matched basis function repetitive control

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ABSTRACT

Many spacecraft suffer from jitter produced by periodic vibration sources such as momentum wheels, reaction wheels, or control moment gyros. Vibration isolation mounts are needed for fine pointing equipment. Active control methods directly addressing frequencies of interest have the potential to completely cancel the influence of these disturbances. Typical repetitive control methods initially address all frequencies of a given period. Matched basis function repetitive control individually addresses each frequency, finding error components at these frequencies using the projection algorithm, and can converge to zero error, using only frequency response knowledge at addressed frequencies. This results in linear control laws but with periodic coefficients. Frequency domain raising produces a time invariant pole/zero model of the control law. A small gain stability theory is developed, that exhibits very strong stability robustness properties to model error. For convergence to zero tracking error it needs only knowledge of the phase response at addressed frequencies, and it must be known within an accuracy of $\pm 90^{\circ}$. Controllers are then designed by pole-zero placement, bypassing the complexity of original periodic coefficient equations. Compared to the usual repetitive control approaches, the approach here eliminates the need for a robustifying zero phase low pass filter, eliminates the need for interpolation in data, and handles multiple unrelated frequencies easily and naturally.

1. Introduction

Many spacecraft suffer from jitter, vibrations produced by internal moving parts. These include slight imbalance in cryogenic pumps, in momentum wheels used to stiffen the attitude dynamics, three reaction wheels often used as actuators in the attitude control system, or four control moment gyros (CMGs) used for the same purpose. Jitter adversely affects fine pointing instruments on board. A common approach for active vibration isolation of such equipment is the filtered *x*-LMS or Multiple Error LMS, Refs. [1–3]. The approach requires a sensor giving a disturbance correlated signal and employs a real time adaptation of a finite impulse response filter that aims to model the needed transfer function from actuator to controlled location. Refs. [2,3] perform experiments on a spacecraft testbed using various different algorithms for jitter control. The testbed employs a 6 degree-of-freedom Stewart platform for vibration isolation, and geophone sensors. This same platform design has been used on orbit. This paper presents a new control approach for active jitter control. Refs. [4,5] perform experiments testing different control algorithms on a spacecraft testbed. Ref. [5] tests the method developed here on a fully floated spacecraft testbed employing a functioning attitude control system using CMG actuators and star tracker feedback. These experiments relate to the use of laser communication

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between satellites which offers substantial advantages in bandwidth and power consumption for transmission of data over long distances. The control algorithms correct for jitter in the control of pan and tilt of the laser steering mirrors. This reference serves as a companion paper giving spacecraft hardware experiments with the algorithm developed here.

Repetitive control (RC), sometimes called repetitive learning control, is a relatively new field that usually aims to eliminate the influence of periodic disturbances on a feedback control system, or it can aim for zero tracking error performing a periodic command, or both. See Refs. [6–10], and for the author's preferred design approaches see Refs. [11–13]. These methods assume one has a method of staying synchronized with the disturbance signal, for example, in Ref. [14], an index pulse is produced each rotation of the disturbance source. This is in fact easy to do for reaction wheels or CMGs based on the phase of the voltage supplied. Unlike the filtered *x*-LMS or Multiple Error LMS approaches, RC initially addresses all frequencies of the same period, i.e., the fundamental of the given period, DC or constant signals, and all harmonics up to Nyquist frequency. Because one usually cannot have a good model of the system all the way to Nyquist frequency, one normally needs to use a zero-phase cutoff filter to cut off the learning at high frequency when model error is too large for convergence. This is done with the *Q* filter of Ref. [10] or the enhanced filters discussed in Refs. [12,13].

RC methods normally only address one period, so in spacecraft applications they can apply to disturbance environments such as a cryogenic pump or a momentum wheel. Refs. [15–20] present repetitive control methods that address multiple unrelated periods, as would be needed to handle disturbances from imbalance in three reaction wheels or four CMGs. Robustness to model error deteriorates as more periods are included (Ref. [20]). These methods address all harmonics of each period, and one can introduce a zero-phase low-pass filter to cut off the learning above some frequency when model error becomes too large for convergence.

Various iterative learning control and repetitive control approaches make use of the concept of basis functions, as in Refs. [21–24]. The most useful basis functions are simple sine and cosine functions of the frequencies of interest. Ref. [25] through [30] present matched basis function repetitive control (MBFRC), which uses the projection algorithm commonly applied in adaptive control (Ref. [31]) to obtain the components of the error on sines and cosines of the frequencies of interest, and applies sine and cosine modifications to the system input that include adjustment for the amplitude and phase change going through the system in order to have the output error be canceled. These adjustments define the matched basis functions, matching input sinusoids to the feedback control system to their resulting control system output sinusoids. As in other forms of RC, an integration is included to create convergence to zero error at the addressed frequencies in spite of substantial model error. Ref. [27] reports experimental tests on the same platform as used in Ref. [2].

Multiple period RC and MBFRC each have their own potential advantages. Multiple period RC simultaneously addresses all frequencies of the periods considered until one cuts out high frequencies with a cutoff filter. MBFRC on the other hand, introduces a separate RC controller for each frequency to be addressed, requiring many controllers for many harmonics. If there are many harmonics that need to be addressed the former approach has an advantage. In exchange for the added complexity of one controller for each frequency, the problem of robustness to high frequency model error is alleviated when using MBFRC. One expects potential improvement in the waterbed effect using MBFRC by allowing high frequencies, above the desired cutoff, to absorb some of the required amplification. An advantage of MBFRC is that multiple unrelated frequencies are addressed in a simple manner, without the complexity needed in Refs. [16] through [20]. Yet another advantage relates to interpolation. The usual RC approaches require interpolation when the period of the addressed frequency is not an integer number of time steps (Ref. [32]). And the interpolation deteriorates at high frequencies. In MBFRC, the basis functions at each frequency allow one to “interpolate” with the actual frequency function of interest.

MBFRC projects the error onto sinusoids and then applies the matched sinusoids to the system. This results in linear equations but with periodic coefficients. Refs. [25,26,28] use Floquet theory, or time domain raising, to study stability when the frequencies of interest have periods that are integer multiples of the sampling time interval. Under the same assumption, Ref. [29] developed stability analysis using the frequency raising technique. A very interesting result of this approach is that the controller involving linear equations involving periodic coefficients related to the basis functions, is seen to have a linear time invariant equivalent model.

The purpose of this paper is to use the time invariant repetitive controller representation to develop very general and simple small gain stability robustness results. The result is obtained by using the departure angle condition from the theory of root locus plots. This approach was used previously to study the simplest form of RC in Ref. [33]. It represents a strong robustness result guaranteeing convergence to zero tracking error for all sufficiently small gain, provided the phase information about your system response for each addressed frequency is accurate to within $\pm 90^\circ$. The result is independent of the system behavior at any other frequency. An additional bonus for the design method presented here is that it no longer requires that the frequencies being addressed have periods that are integer multiples of the sample time interval, or use interpolation.

In the next sections, first the MBFRC algorithm is presented, then the equivalent time invariant control laws for each frequency. Then the case of addressing one frequency only is treated to determine the departure angles from poles on the unit circle, which is then generalized to apply to any number of addressed frequencies. Numerical examples are presented.

2. The matched basis function repetitive control algorithm

This section summarizes the MBFRC algorithm. Usually the RC controller adjusts the command to a feedback control system, although it could simply be adjusting the input to