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Two-dimensional over-expanded jet flow parameters in supersonic nozzle lip vicinity

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ABSTRACT

The mathematical model for two-dimensional (plane or axis-symmetric) over-expanded jet flow parameters analysis in the vicinity of supersonic nozzle lip is proposed. The variation of the key parameters of this problem (e.g., the geometrical curvature of oblique shock emanating from the nozzle edge) is studied parametrically depending of jet flow parameters, such as Mach number, jet incalculability, and the ratio of gas specific heats. It was proved that differential parameters of the flow field crucially depend not only of the key parameters, but on the symmetry type as well.

1. Introduction

The effectiveness of the aviation jet engines and rocket propulsions can be achieved in many respects due to correct nozzle configuration and supersonic jet flow control. It is important to control the shock-wave configurations in under-expanded, correctly expanded or over-expanded jets flowing out of the nozzle to avoid the boundary layer separation, auto-oscillating regimes, and longitudinal instability of the flow supplying the reactive force.

Differential characteristics of the supersonic flow filed in the vicinity of the nozzle edge often relate to such physical effects as Taylor–Görtler instability, regular/Mach reflection mutual transition at small Mach numbers, self-oscillation phenomena in free, submerged and impact jets.

This article presents a fragment of a complex study on supersonic jet flows in a vicinity of a nozzle edge. Differential conditions of dynamic coexistence [1] are applied to gas dynamic variables and their spatial derivatives at both sides of oblique shock waves emanating from an edge of two-dimensional (plane or axis-symmetric) over-expanded jet flowing into submerged media. Isobaricity condition at the inviscid gas jet boundary allows us to investigate a change of the shock geometry, and of the jet flow boundary, finding special and extreme cases of the emanation. A crucial differential characteristic which allows defining all main flow non-uniformities in the edge vicinity is the geometrical curvature of the oblique shock emanating from the nozzle lip. The article presents analysis of variations of the differential characteristic in a two-dimensional jet of a non-viscous perfect gas in relation to the outflow conditions.

2. Governing relations

The shock wave AT (Fig. 1) emanating from the edge A of the supersonic nozzle with θ opening angle has the strength (intensity) $J=1/n$ where $n=p_a/p_n n$ is jet incalculability determined by comparison between static pressure p_a of the emanating jet in the nozzle edge vicinity and the surrounding pressure p_n.

The intensity J (relation between pressures behind and ahead of a shock wave [2]) is limited in the range $1<J<J_m$ where

$$J_m=(1+\varepsilon)M^2-\varepsilon$$

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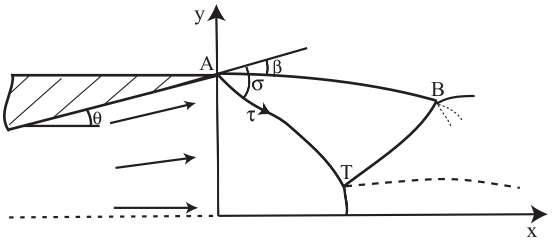

Fig. 1. Scheme of the over-expanded jet flow into ambient gas media.

is the strength of a direct shock wave in the flow with the Mach number M ahead of it, M is flow Mach number in the vicinity of point A upstream the shock, $\varepsilon=(\gamma-1)/(\gamma+1)$, and γ is the ratio of gas specific heats (it is assumed in the further calculations that $\gamma=1.4$).

Flow deflection angle β at the shock wave relates to its intensity and the Mach number ahead of it as follows:

$$\mathrm{tg}|\beta|=\sqrt{\frac{J_m-J}{J+\varepsilon}}\frac{(1-\varepsilon)(J-1)}{J_m+\varepsilon-(1-\varepsilon)(J-1)}. \quad (1)$$

Shock slope angle σ to the flow velocity vector ahead of the shock and the flow Mach number M_2 downstream the shock wave are related to M and J as follows:

$$J=(1+\varepsilon)M^2\sin^2\sigma-\varepsilon, \quad (2)$$

$$M_2=\sqrt{\frac{(J+\varepsilon)M^2-(1-\varepsilon)(J^2-1)}{J(1+\varepsilon J)}}. \quad (3)$$

In a general case, spatial derivatives of various jet parameters undergo a break at the shock wave surface, as well as flow parameters themselves. The variations of the spatial derivatives on the shock sides are described by differential conditions of dynamic coexistence [1] in the following form

$$N_{i2}=C_i\sum_{j=1}^{5}A_{ij}N_j, i=1\ldots3, \quad (4)$$

where N_{i2} are flow non-uniformities behind the shock wave, N_j are flow non-uniformities ahead of the shock; C_i and A_{ij} are the factors which depend on M, J and θ. The non-uniformities $N_1=\partial\ln p/\partial s$, $N_2=\partial\theta/\partial s$, and $N_3=\partial\ln p_0/\partial n$ characterize, correspondingly: flow non-isobaricity, streamline curvature and the gradient of the total pressure in isoenergetic flow; $N_4=\delta/y$ is symmetry type factor ($\delta=0$in plane flow, and $\delta=1$ in axis-symmetric one); $N_5\equiv K_\sigma$ is the own geometrical curvature of the shock. Conditions (4) determine, in particular, the flow non-uniformities in the compressed layer directly behind the shock wave of the known curvature, if the flow field ahead of it is known.

The writing (4) of the differential conditions on the stationary shocks in steady non-uniform flow is certainly not unique. One of the most modern forms of differential flow field parameters mutual dependence on shock sides was deduced in Ref. [3] and applied later [4] for gas entropy variation and flow vorticity analysis. The results reached below and elaborated in Refs. [5,6] for plane over-expanded jet are independent of form of writing of correctly deduced differential relations on stationary shock.

Condition of flow isobaricity ($N_{12}=0$) along the jet boundary AB (Fig. 1) determines a sought shock wave curvature

$$K_\sigma=-\sum_{j=1}^{4}A_{1j}N_j/A_{15}, \quad (5)$$

as well as other differential flow field parameters in the compressed layer immediately behind the shock.

In particular, jet boundary curvature ($N_{22}\equiv K_\tau$) in point A depends on it as follows:

$$K_\tau=C_2\sum_{j=1}^{4}(A_{2j}A_{15}-A_{1j}A_{25})N_j/A_{15}, \quad (6)$$

According to Refs. [7–10], K_τ determines formation and development of the Taylor–Görtler longitudinal instability.

Relation (5), two-dimensional perfect gas flow equations applied in front of the shock wave and behind it in natural coordinates (s, n)

$$\frac{M^2-1}{\gamma M^2}N_1+\frac{\partial\theta}{\partial n}+N_4\sin\theta=0, \gamma M^2N_2=-\frac{\partial\ln p}{\partial n}, \frac{\partial p_0}{\partial s}=0,$$

and relations ((1)–(3)) between the shock wave shape, its intensity and the Mach number on the shock wave sides determine, after some transformations, e.g., local changes in the intensity and the Mach number behind the shock wave in τ direction along the shock wave

$$\frac{dJ}{d\tau}=-2(J+\varepsilon)(B_1N_1+B_2N_2+B_3N_3+\chi acN_4\sin\theta+qK_\sigma),$$
$$\frac{dM_2}{d\tau}=-\left[1+\varepsilon(M_2^2-1)\right]\times\left(\frac{M_2N_{22}}{1-\varepsilon}+\frac{N_{32}}{(1+\varepsilon)M_2}\right)\times\sin(\sigma-\beta). \quad (7)$$

Here the direction index $\chi=-1$ relates to the incident shock wave at Fig. 1, $c=\sqrt{(J+\varepsilon)/(J_m+\varepsilon)}$, $q=\sqrt{(J_m-J)/(J+\varepsilon)}$, and factors B_i have the following form:

$$B_1=\chi ac\times\frac{1-(1-2\varepsilon)(M^2-1)}{(1+\varepsilon)M^2},$$
$$B_2=c\times\left(\frac{1+\varepsilon\left(M^2-1\right)}{1-\varepsilon}-q^2\right),\quad B_3=c\times\frac{1+\varepsilon(M^2-1)}{J_m+\varepsilon}.$$

Total pressure preservation coefficient at the shock is $I=p_{02}/p_0=(JE^\gamma)^{(1-\varepsilon)/2\varepsilon}$.

Here p_0 and p_{02} are flow stagnation pressures before the shock wave and behind it, $E=\rho_0/\rho_{02}=(1+\varepsilon J)/(J+\varepsilon)$ is the inverse ratio of gas densities at shock wave sides, ΔS is entropy variation

$$\Delta S=c_v\ln(JE^\gamma)$$

(c_v is gas specific heat at constant volume). All these parameters are in uniform dependence on shock intensity. Thus, the non-uniformity (flow vorticity) N_{32}, according to (4), and the direction of an isoenergetic jet velocity vortex vector, according to Crocco formula, are determined by