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The motion of the thread with a variable length

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ABSTRACT

This paper is devoted to the peculiarities of a process of inertial tether system deployment of the great length (tens of kilometers). And one of the main practical issues of this article is the behavior of maximum oscillation amplitude.

First of all the exact analytical solution of the original problem is received. Furthermore it is shown that ratio of transverse wave velocity of the tether to the velocity of deployment is one of the main parameters of the problem. In limiting case, when initial tether length equals zero, the solution is received using another method – the method of progressing waves. Also it is demonstrated that the limiting case agrees with the previously received results.

Analysis of the solution has revealed that the maximum oscillation amplitude of the tether almost always rises with the time. This effect corresponds to asymptotic instability of the solution (in this article instability is regarded as rising of oscillation amplitude with the time). Only in specific degenerate conditions the solution is stable. The received results of amplitude increase have to be considered in simulation of the tether system behavior as far as small perturbative vibrations at the reel system always take place.

1. Introduction

Traditionally in practical applications flexible coupling in ways of threads, cables and tethers is widely used. At the present time attempts of using tether systems in space are made. Furthermore for the last 5 years a great number of experiments have been run to analyze the behavior of such systems, retrieve data and visualize the considered process. For example tethered Satellite Systems TSS1 and TSS1-R as well as Oedipus, SEDS 1 and SEDS 2 (Small Expendable Deployer System), YES and YES2 (Young Engineers' Satellite) and other missions have been successfully launched.

For realization of tether deployment, which has the great length, new high-test devices have been developed. The descriptions of such mechanisms can be found, for example, in [1]. Furthermore such devices have been used for deployment and following reeling of the tether for the purpose of making artificial gravity during MARS-g project. As a matter of fact these technologies can be used in the prospective missions to Mars which are now planned by NASA and ESA. For receiving of new information and data as well as for debugging of devices and tether systems a great number of starts have been needed to be made. For that purpose new systems have been developed which have been a payload on board of small satellites. As a result of these tendencies new ruggedized devices with a low mass have been produced. For example, during DELFI-1 project mechanism of the same type has been used for deployment of aluminum thread with the length of 1 km. Information about this project can be found in [2].

Nowadays there are many projects in which tether systems are highly used. They deal with, for example, "space elevators", cargo delivery from space without usage of ferry ships ("space mail") or removal of space debris. Space debris due to its annual growth has become a serious hazard to long term space programs [3]. For minimization of a number of geo-based space debris objects in the paper [4] detailed analysis has been made and various scenarios and cases have been considered. Currently there are two main tendencies in solving space debris problem. Firstly, that is the development of the shield for space crafts from a high velocity impact of space debris elements. The model that acceptably predicts

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the consequences of such interaction is represented in the work [5]. It is based on the received closed form solution that determines the high velocity fragment interaction with thin walled fluid-filled containments. And returning to the another tendency, that is the active debris removal (ADR), it is important to mention that the growth of the LEO (Low Earth Orbit) debris population has been shown even in spite of the 25-year post-mission disposal (PMD) rule. In [6] it is suggested that additional removal of a small number of certain debris objects per year can stabilize the situation in the future. The following studies have tried to concretize the previous research. For example, in [7] the influence of four main parameters (launch, explosion rates, magnitude of solar activity and level of PMD compliance) on the LEO debris population has been investigated. Moreover for establishing of the effectiveness of this method different ADR rates have been considered such as zero, five and ten removals per year.

As for demonstration and development of potential cargo delivery from space with the use of the tether system in September 14, 2007 mission YES2 has been launched. During this mission deployment of the tether with the length of 32 km has been made and therefore the capsule with the weight of 6 kg has been disorbited. As a result the information about dynamical processes which are taking place in the tether during its deployment has been received that is capsule coordinates, angle of deviation from the vertical, velocity of the deployment and length of the tether plots against time have been constructed [8–9]. Furthermore tether tension jump data has been obtained. This discontinuous jump takes place immediately after the end of the deployment as a result of interaction of lateral and longitudinal disturbances with the borders [8]. Moreover mathematical modeling of the considered process has been undertaken. And with the use of data which has been received during YES2 mission the verifying of the developed model has been made. As a result conducted analysis has shown a quick response of the process to its initial parameters such as the capsule's weight or its initial velocity at the moment of undocking. After all it has revealed the importance of taking into account at the last stage of the deployment the effects which are connected with propagation of sonic waves in tether and their satellite and capsule reflection [9–11].

The foregoing shows the relevance of analysis of tether deployment various aspects. In this paper a process of deployment has been simulated in assumption that tether tension remains constant. We considered transverse modes propagation in a homogeneous ideal thread with variable length [12]. Within this model the following problem has been stated and solved. That is from the state of rest tether length begins to increase with a constant speed. And herewith one tether end is fixed, the other oscillates in a predetermined law. The received analytical solution has been used for a motion study and as well as investigation of maximum of oscillation amplitude behavior with the time.

2. Mathematical model of the tether deployment from a reel

Let us consider motions of a homogeneous perfectly flexible thread with a variable length in a plane (x, y) (Fig. 1). Also we assign the corresponding arc length in

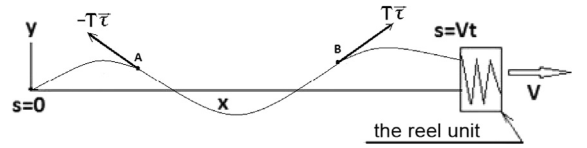

Fig. 1. General schema of motion. One tether end ($s=0$) is fixed. In a process of dereeler motion with the velocity V_0 tether length is increasing from the initial length L_0 to the actual length L_0+V_0t. In addition the running end of the thread ($s=L_0+V_0t$) is staggering lateral modes with a small amplitude $y=a\sin\omega t$.

the unextended state to each material point of the fiber. As widely known a tensile force $\mathbf{T}=T\boldsymbol{\tau}$ for an ideal thread is tangential to it. Let τ be the unit vector which is tangential to the tether. Then the tensile force vector can be represented as $\mathbf{T}=T\boldsymbol{\tau}$, where T is the tension. In general case of a thread's motion it needs to determine a position vector $\boldsymbol{r}$, tension T, strain ε, unit tangent vector $\boldsymbol{\tau}$ for each material point s at every instant. Therefore the sought quantities are functions of two variables (s,t). Let us introduce components $v_x(s,t)$, $v_y(s,t)$ of velocity vector v of points of the thread, initial linear density ρ and body forces density g.

The law of variation of momentum can be stated for every designated segment AB (Fig. 1)

$$\frac{\partial}{\partial t}\int_{s_A}^{s_B}\rho\mathbf{v}ds=\int_{s_A}^{s_B}\left[\frac{\partial(T\boldsymbol{\tau})}{\partial s}+\rho\mathbf{g}\right]ds \text{ or}$$
$$\int_{s_A}^{s_B}\left[\rho\frac{\partial\mathbf{v}}{\partial t}-\frac{\partial(T\boldsymbol{\tau})}{\partial s}-\rho\mathbf{g}\right]ds=0.$$

For the case of continuous motion and due to arbitrariness of A,B the motion equation of the thread follows from the last equality

$$\frac{\partial\mathbf{v}}{\partial t}=\frac{1}{\rho}\frac{\partial(T\boldsymbol{\tau})}{\partial s}+\mathbf{g}.$$

Let us consider that the tether tension is close to invariable T_0 during the process of thread deployment with a constant speed. The value of this tension depends on the speed of deployment and also is predicated upon technical characteristics of device. We shall also assume that body forces are small to negligible and in the process of motion the angle between tangency and axis x as well as the longitudinal velocity of points are small values. In that case only transverse motion is regarded. Besides from the smallness of required values it follows that $\tau_y\approx\partial y/\partial s$, $s\approx x$. Taking into account the expression for transverse velocity we receive the next equation

$$\frac{\partial^2 y}{\partial t^2}=b^2\frac{\partial^2 y}{\partial x^2},\ b^2=\frac{T_0}{\rho}. \quad (1)$$

According to the suggested model of the deployment (Fig. 1) Eq. (1) can be completed with the following initial and boundary conditions:

$$t=0,\ y(x,0)=0,\ \frac{\partial y}{\partial t}(x,0)=0;\ x=0,\ y(0,t)=0;$$
$$x=L_0+V_0t,\ y=a\sin\omega t. \quad (2)$$

Let us turn to dimensionless values

$$\tilde{y}=\frac{y}{L_0},\ \tilde{x}=\frac{x}{L_0},\ \tilde{t}=\frac{V_0t}{L_0},\ \tilde{b}=\frac{b}{V_0},\ \tilde{\omega}=\frac{\omega L_0}{V_0},\ \tilde{a}=\frac{a}{L_0},$$