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Long-term evolution of Galileo operational orbits by canonical perturbation theory ☆

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ABSTRACT

Galileo operational orbits are slightly affected by the 3 to 5 tesseral resonance, an effect that can be much more important in the case of disposal orbits. Proceeding by canonical perturbation theory we show that the part of the long-term Hamiltonian corresponding to the non-centralities of the Earth's gravitational potential can be replaced by an *intermediary* that shows the pendulum dynamics of the 3 to 5 tesseral resonance problem. Inclusion of lunisolar perturbations requires a semi-analytical integration, which is compared with the corresponding results from the well-established Draper Semi-analytical Satellite Theory.

1. Introduction

The Galileo constellation is designed such that the satellites move in almost-circular MEO orbits at an altitude of about 23,222 km over the surface of the Earth and at 56° of nominal inclination[1]; this orbital configuration corresponds to a 17 to 10 repeat groundtrack condition (Galileo satellites will complete 17 nodal orbits while the Earth completes 10 rotations). If we test the Galileo operational orbit for the presence of tesseral resonance using a 30 × 30 geopotential model, we see that order 12, order 17, and order 29 terms may generate resonant effects.[2] But these terms are multiplied by the parallax factor raised to the order of the resonant term. Here the parallax factor is the ratio of the radius of the Earth divided by the mean semi-major axis of the orbit. These powers of the parallax factor are quite small for the resonant terms in the Galileo operational orbit. Hence, effects originating from the shallow Galileo tesseral resonance are not expected to be observed. However the Galileo disposal orbits may be a few hundred kilometers above the operational orbits. The Galileo disposal orbits are close to the 5 to 3 commensurability with the Earth's rotation period and, therefore, may be non-negligibly affected by tesseral resonances due to the fifth-order, tenth-order, and fifteenth-order tesseral harmonics.

The shallow resonant Galileo operational orbits contrast significantly with the GPS operational orbits which are in deep resonance with the even order harmonics in the geopotential [1].[3] Besides the J_2 perturbation, the most important disturbing effect at the altitude of Galileo satellites is due to lunar and solar gravitational attraction.

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[1] http://www.esa.int/esaNA/galileo.html

[2] By resonance, we terms with periods greater than 10 days.

[3] A mathematical definition of deep and shallow resonances is given, for instance, in [2].

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These destabilizing effects—lunisolar perturbations and tesseral resonances—which are suffered by all classical GNSS constellations, together with the growing population of satellites in the MEO region, raise concerns for the orbit evolution of operational satellites in medium and high earth orbits as well as debris. [3–9].

In the case of operational orbits, much attention has been paid to tesseral resonances, which may introduce undesired long-periodic variations in the semimajor axis. But the study of lunisolar resonances [10,11] is also of definitive importance in the very long-term scales in which debris evolve. Lunisolar resonances are commonly identified using a basic model that includes the first-order long-term effects of the Earth's oblateness and of the Sun and Moon, which are assumed to move in distant circular orbits about the Earth in the fixed Ecliptic plane (see Ref. [12] and references therein).

The long-term evolution of both the operational orbits and debris can be studied by means of purely numerical techniques [5–8]. However, efficient numerical integration is still computationally expensive for long term propagations which makes the search for alternative approaches desirable. In this respect, the analytical filtering of short-period terms from the equations of motion is a useful artifice in the development of either symplectic [13–16] or semi-analytical integrators [17,18,9].[4]

Analytical theories are also of interest, although they may be constrained in their application due to the problem of singularities caused by resonances, as well as modeling non-conservative perturbations. Nevertheless, they are extensively used [19–21] and the canonical perturbation theory has proved to be a useful tool in disclosing the dynamical characteristics of the artificial satellite problem, such as frozen orbits distribution [22], or in the search for intermediary orbits [23,24,1].

We use canonical perturbation theory by Lie transforms to approach the long-term evolution of Galileo orbits. We deal with a three-degrees-of-freedom, time-dependent Hamiltonian which is made up from a Keplerian part perturbed by the non-centralities of the geopotential as well as lunar and solar attraction. The averaging of the short-period terms reduces the number of degrees of freedom by one. Because the Galileo orbits are close to the 5 to 3 resonance, we do not average corresponding resonant terms of the geopotential.

Concerning the geopotential, the high altitude of Galileo satellites limits the number of significant coefficients in the Earth's gravity field. In this theory, the geopotential is assumed to be a 8×8 tesseral model. After averaging the short-period terms, this part of the Hamiltonian takes the form of an intermediary, plus a perturbation which is factored by the eccentricity. Hence, since the Galileo orbits are designed to have very low eccentricities, these operational orbits accept an analytical solution of the tesserals-only problem.

However, lunisolar effects are quite important at the altitude of Galileo satellites, driving secular variations in eccentricity and long-period variations in inclination [25,26]. These effects are more apparent in the case of disposal orbits or debris. Therefore, the corresponding time-dependent terms must be retained in the Hamiltonian, whose long-term evolution must be integrated semi-analytically. Nevertheless, since the averaged equations only depend on very slow evolving angles, the semi-analytical integration is fast and efficient.

The analytical and semi-analytical results are validated with the Draper Semi-analytical Satellite Theory [17] (DSST). Because of the very low eccentricities of Galileo orbits, the canonical perturbation theory has been computed in non-singular elements instead of the classical approach in Delaunay elements. We find an encouraging agreement between results of the canonical perturbation theory and those of DSST.

2. Model

The long term evolution of Earth's artificial satellites is influenced by different effects that slightly distort the Keplerian solution resulting from the Earth's central attraction. In the case of satellites in the MEO region, the non-centrality of the Earth's gravitational potential and the gravitational pull of the Moon and the Sun are the most important effects. Solar radiation pressure also affects the orbits of Galileo satellites, and an error of 10% on the effective area to mass ratio has been claimed to introduce an error of 0.3° in argument of latitude after 12 years of propagation [27]. However, the other orbital elements are barely affected, except in the case of the extremely high area-to-mass ratio debris originated from breakup fragments [6,7].[5]

Then, we choose a model in which the perturbed Keplerian motion of a massless satellite about the Earth is derived from the potential

$$\mathcal{V} = -\frac{\mu}{r} + V + V' \qquad (1)$$

where μ is the Earth's gravitational parameter, r is the distance from the origin, the disturbing potential V represents the non-centralities of the geopotential due to the Earth's figure effects, and V' is the third-body disturbing potential.

In an Earth-centered, Earth-fixed coordinate system, the non-centralities of the Earth's gravitational field are derived from the potential

$$V = -\frac{\mu}{r}\sum_{j\geq 2}\left(\frac{\alpha}{r}\right)^j \left[C_{j,0}P_{j,0}(\sin\varphi). + \sum_{k=1}^{j}(C_{j,k}\cos k\vartheta + S_{j,k}\sin k\vartheta)P_{j,k}(\sin\varphi)\right] \qquad (2)$$

[4] Symplectic integration aims at preserving the geometric properties of the problem during very long time-scales, as, for instance, in the search for chaotic motion. It sacrifices precision in favor of velocity and hence is based on very simple integration schemes. On the contrary, semi-analytical integration is used in the computation of precise ephemeris. It works in shorter time-scales in which numerical artifacts produced by the accumulation of truncation errors may be avoided by means of standard numerical integration schemes.

[5] Other unique orbits that have large SRP effects in elements other than the argument of latitude are sun-synchronous, retrograde critical inclination, and repeat-ground track.