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Three dimensional temperature field in a conducting sphere due to an arbitrarily located split ring heating source

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ABSTRACT

Thermal control of spacecrafts plays an important role in space missions. In the design stage the preliminary thermal analysis of the spacecraft requires an estimate of the conductive thermal resistance between the various spacecraft components. With this in mind, the fully three dimensional problem of determining the thermal field in a conducting sphere with an asymmetric split ring current carrying heating source is resolved in an analytical or almost analytical form, implying either a closed form solution or utmost expressions involving a simple numerical integration. This has immediate application for evaluation of thermal resistance in spacecrafts. Green's function integral techniques are used. Comparisons are made with series solutions and also with purely numerical solutions to contrast the simplicity and highlight the elegance of the present method. Parametric studies reveal expected behavior.

1. Introduction

Due to the hostile environment of a spacecraft being alternately exposed to blazing sun and the cold of deep space, many spacecrafts are equipped with sophisticated active thermal control systems to ensure that the various components are operating safely. Among others, electrical resistance heaters which are turned on or off depending on the thermal load are often commonly used in all spacecrafts. During the preliminary design stages for thermal analysis it is necessary to supply the data for the thermal conductive resistance between the various components or nodes of the spacecraft [1]. A systematic tabulation of the thermal resistance requires a complete *a priori* knowledge of the temperature field for many configurations. Recently, Smirnov et al. [2] have numerically analyzed the thermal field inside the spherical capsule of the "Foton-M" spacecraft showing the importance of the temperature determination in the thermal design of spacecrafts. Although many complex geometries with complex boundary conditions can be solved by present day numerical computational packages or even "black boxes", still much can be done using analytical or quasi-analytical approaches yielding greater insight than the purely numerical solutions, especially when parametric studies are to be made. Further the analytical solutions provide a validity check for purely numerical solutions. In this paper we present an analytical or almost analytical solution, implying either a closed form solution or utmost expressions involving a simple numerical integration, to the three dimensional problem of determining the thermal field in a conducting sphere with an asymmetric split ring heating source.

The conduction thermal analysis and diffusion of heat have been exhaustively treated by the classic book of Carslaw

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and Jaeger [3] and in an updated notation by Arpaci [4] and more recently by Bejan [5]. Specific problems of application to spacecrafts have been treated by Venkataraman et al. [6], Venkataraman and Egalon [7], Venkataraman et al. [8] and Venkataraman and Meza [9].

Lin [10] has analyzed the explicit full field analytical solutions of heat conduction in two-dimensional composite layer, wedge and circular media using a combined Fourier transform and conformal mapping methods. Jain et al. [11] used a series solution to analytically analyze the transient multilayer heat conduction in two-dimensional spherical (r,θ) coordinates. Kidawa-Kukla [12,13] used the Green's function method to analyze the transient temperature field due to a moving point heat source in an annular plate and due to an oscillating heat stream. Beck [14] presented an exact solution method for transient three dimensional heat conduction problem over a rectangular region on a homogeneous plate using Green's functions. Seremet [15] utilized a new Green's function and a Green-type integral formula to analyze thermo-elastic problem of a quadrant and presented a closed form solution. Yilmazer and Kocar [16] used Green's functions to obtain analytical solutions for the annulus of eccentric spheres with internal heat generation and isothermal boundaries. Woodbury and Beck [17] treated the problem of a planar slab with one end subjected to heat flux increasing as the fourth power of time and the other end insulated, using Green's function techniques. The review of the literature shows that still many interesting practical problems can be solved analytically or quasi-analytically and the problem under consideration has not been treated in an analytical manner before.

2. Problem statement

We consider the problem of determining the steady state temperature distribution of a conducting sphere of radius a and thermal conductivity k with an asymmetric split ring heating element (Fig. 1). The heating element is a thin current carrying conductor, in the form of an infinitesimally thin circular arc of radius b and subtending an angle β at the center of the arc. The ring is in any arbitrary orientation. The plane parallel to the ring at the center of the sphere is taken as the equatorial plane *oxy* and the axis *oz* is perpendicular to the equatorial plane passing though the center of the ring. One end of the ring starts from the x*oz* plane.

Any point on the arc source is at a radial distance c from the origin of the coordinate system. Thus the location of the heating source is completely defined by the radius b of the source and its height h above the equatorial plane. The source heat generation intensity per unit length and per unit time is Λ (watts/m). The surface of the sphere is kept at a constant temperature T_B. This is done to reduce the algebra by eliminating the need to evaluate the second integral of Eq. (5). The method here is applicable even when the temperature of the surface is varying. For preliminary analysis, the spacecraft surface temperature can be approximated by a mean surface temperature. However, in specific problems with mixed boundary conditions involving both the temperature and its gradient at the surface, the method of images used here for determining the Green's function fails.

3. Integral form of the temperature using the Green's function representation

The steady state temperature $T(\vec{x})$ at any point $\vec{x}$ in a body of constant thermal conductivity k with an internal heat generation per unit volume and per unit time of $Q(\vec{x})$, is governed by

$$\nabla^2 T(\vec{x}) = -Q(\vec{x})/k \tag{1}$$

with the appropriate boundary conditions. In our case

$$T(\vec{x}) = T_B \text{ on the boundaries} \tag{2}$$

In what follows, we use Green's function integral method for solving the above equation [8,18,19]. Green's

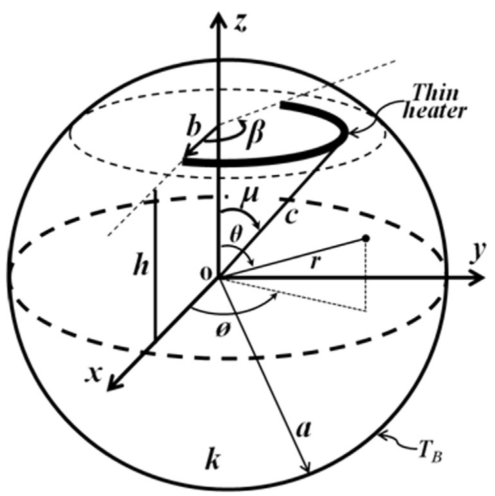

Fig. 1. Conducting sphere with a split ring heating source.

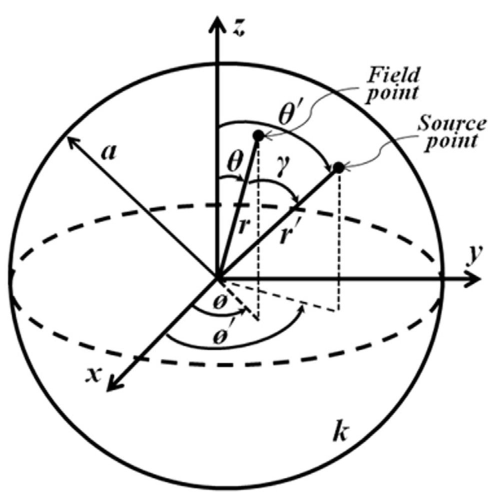

Fig. 2. Source and field points for a sphere of thermal conductivity k.