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Time-optimal trajectory planning for underactuated spacecraft using a hybrid particle swarm optimization algorithm

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ABSTRACT

A hybrid algorithm combining particle swarm optimization (PSO) algorithm with the Legendre pseudospectral method (LPM) is proposed for solving time-optimal trajectory planning problem of underactuated spacecrafts. At the beginning phase of the searching process, an initialization generator is constructed by the PSO algorithm due to its strong global searching ability and robustness to random initial values, however, PSO algorithm has a disadvantage that its convergence rate around the global optimum is slow. Then, when the change in fitness function is smaller than a predefined value, the searching algorithm is switched to the LPM to accelerate the searching process. Thus, with the obtained solutions by the PSO algorithm as a set of proper initial guesses, the hybrid algorithm can find a global optimum more quickly and accurately. 200 Monte Carlo simulations results demonstrate that the proposed hybrid PSO–LPM algorithm has greater advantages in terms of global searching capability and convergence rate than both single PSO algorithm and LPM algorithm. Moreover, the PSO–LPM algorithm is also robust to random initial values.

1. Introduction

The time-optimal trajectory planning problem of rigid spacecrafts has received consistent interest in last decades, since rapid attitude maneuvers are critical to various space missions such as military observation and satellite communication [1]. To solve this problem using classic optimal theory, the Hamilton–Jacobi–Bellman (HJB) partial differential equations must be solved. However, due to the complicated nonlinear dynamics of high dimension and many types of constraints for rigid spacecraft, the HJB equations are very difficult to be solved. Thus, numerical methods base on gradient information have been widely used [2–4]. There are two distinct branches of numerical methods: indirect and direct [5], both of which attempt to minimize the functions and constraint violations using discrete approximations of the parameters of the system. Compared to classic optimal theory, numerical methods have advantages in terms of flexible applicability to practical complex problems and meanwhile guaranteeing a relatively quick convergence rate and very accurate results. However, two key drawbacks inherent to both indirect and direct methods are: (1) the lack of a global search capability; and (2) the requirement of suitable initial guesses [6]. A set of poor initial guesses may cause the optimization program trapped at a local optimum in multimodal problems or even diverge.

To deal with this problem, evolutionary algorithms (EAs) such as genetic algorithms (GA), differential evolution (DE) and particle swarm optimization (PSO) have been employed. These algorithms usually have better ability to converge to a global optimum or a near optimum solution than traditional optimization methods, and moreover, they are not sensitive to initial guesses of solutions. Thus, EAs have been successfully applied in many real world nonlinear optimal problems [6–9]. However, as we know, EAs are characterized by their poor numerical accuracy and difficult constraint handling.

In recent years, in order to overcome these shortages of both numerical methods and EAs, a series of hybrid optimization algorithms have been proposed [10–14]. The fundamental idea of these algorithms is that first an EA with random initial solutions is utilized to enhance the

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global search capability. When the change in fitness value is smaller than a predefined value, or the candidate solution of GA or the particles in swarm being close to the global optimum [13–15], the evolution process is stopped generating. Then, the searching algorithm is switched to a direct method, which can achieve a faster convergence rate around the global optimum and a higher accuracy than the EA. And, the obtained near-optimal solution by EA will be taken as an initial guess of solutions for the subsequent nonlinear programming (NLP) solver. In this way the hybrid algorithm may find a global optimal solution more quickly and accurately.

Other than the literature on hybrid optimization algorithms with GA as a starter engine [13,14], in this paper, a new hybrid algorithm is proposed which combines the PSO algorithm and the Legendre pseudospectral method (LPM) for the time-optimal trajectory planning problem of an underactuated spacecraft. Recently, PSO algorithm has been found to be a promising technique for a variety of optimization problems due to its superior advantages. PSO is initialized with a population of random solutions and searches for the optimum by updating generations. And compared to GA and DE, PSO has no operators such as crossover, mutation and selection, and it has fewer parameters to adjust. Therefore, PSO is quite easy to implement and it also can be applied in many areas to replace GA. In this present work LPM is used as a discretization scheme, and PSO is used to find an initial starting point being within the domain of convergence of the particular NLP solvers. And further, the differential flatness property of the spacecraft is also discussed, which can reduce the number of variables to be optimized in order to decrease the time consumption of the whole optimization process.

The rest of the paper is organized as follows. Section 2 describes the formulation of the trajectory optimization problem. Section 3 discusses the flatness property of a general nonlinear system. In Section 4 the hybrid PSO-LPM algorithm is introduced. Section 5 applies the proposed algorithm to a minimum-time trajectory planning problem for a rigid spacecraft under actuator failure condition, and presents the simulation results. Finally, conclusion is contained in Section 6.

2. Problem formulation

The purpose of trajectory optimization is to determine the time history of a control vector $\boldsymbol{u}(\tau), \tau\in[\tau_0, \tau_f]$ with considering the constraints, in order to minimize the cost function J. Typically, J can be written in Bolza form consisting a Mayer term Φ and a Lagrange term L [5]:

$$J = \Phi(\boldsymbol{x}(\tau_f), \tau_f) + \int_{\tau_0}^{\tau_f} L(\boldsymbol{x}(\tau), \boldsymbol{u}(\tau), \tau)\mathrm{d}\tau \tag{1}$$

Subject to

(1) State constraints

$$\dot{\boldsymbol{x}}(\tau) = \boldsymbol{f}(\boldsymbol{x}(\tau), \boldsymbol{u}(\tau), \tau) \tag{2}$$

(2) Endpoint constraints with upper and lower bounds on function $\boldsymbol{\psi}[\cdot]$

$$\boldsymbol{\psi}_{0l} \le \boldsymbol{\psi}(\boldsymbol{x}(\tau_0), \boldsymbol{u}(\tau_0), \tau_0) \le \boldsymbol{\psi}_{0u}, \quad \boldsymbol{\psi}_{fl} \le \boldsymbol{\psi}(\boldsymbol{x}(\tau_f), \boldsymbol{u}(\tau_f), \tau_f) \le \boldsymbol{\psi}_{fu} \tag{3}$$

(3) Path constraints with upper and lower bounds on function $\boldsymbol{g}[\cdot]$

$$\boldsymbol{g}_l \le \boldsymbol{g}(\boldsymbol{x}(\tau),\ \boldsymbol{u}(\tau), \tau) \le \boldsymbol{g}_u \tag{4}$$

(4) Box constraints on states and controls

$$\boldsymbol{x}_l \le \boldsymbol{x}(\tau) \le \boldsymbol{x}_u, \quad \boldsymbol{u}_l \le \boldsymbol{u}(\tau) \le \boldsymbol{u}_u \tag{5}$$

In Eqs. (1)–(5), $\boldsymbol{x}(\tau)\in\mathbf{R}^n$ is the state vector of the system, and $\boldsymbol{u}(\tau)\in\mathbf{R}^m$ is the control vector.

3. Problem conversion

3.1. Notation of differential flatness

A dynamic system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \quad \boldsymbol{x}\in\mathbf{R}^n,\ \boldsymbol{u}\in\mathbf{R}^m \tag{6}$$

is differentially flat or just flat, if there exist smooth maps $\boldsymbol{Y}$, $\boldsymbol{A}$ and $\boldsymbol{B}$ defining on open neighborhoods of $\mathbf{R}^n \times (\mathbf{R}^m)^{\rho+1}$, $(\mathbf{R}^m)^{r+1}$ and $(\mathbf{R}^m)^{r+2}$, such that

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{Y}(\boldsymbol{x}, \boldsymbol{u}, \dot{\boldsymbol{u}}, \ddot{\boldsymbol{u}}, \dots, \boldsymbol{u}^{(\rho)}) \\ \boldsymbol{x} &= \boldsymbol{A}(\boldsymbol{y}, \dot{\boldsymbol{y}}, \ddot{\boldsymbol{y}}, \dots, \boldsymbol{y}^{(r)}) \\ \boldsymbol{u} &= \boldsymbol{B}(\boldsymbol{y}, \dot{\boldsymbol{y}}, \ddot{\boldsymbol{y}}, \dots, \boldsymbol{y}^{(r+1)}) \end{aligned} \tag{7}$$

here ρ and r are positive integers, $\boldsymbol{y}$ is called a set of flat outputs, and the components of $\boldsymbol{y}$ are not related by a differential relation [16].

3.2. Problem reformulation in flat output space

From the notation of flatness, it appears that if a dynamic system is flat, then its state and input variables can be parameterized in terms of the set of flat outputs and a finite number of their derivatives. Thus, the above original optimal problem can be converted and reformulated in flat output space as

$$\begin{aligned} &\min_{\tilde{\boldsymbol{y}}(\tau)} \tilde{J}(\tilde{\boldsymbol{y}}(\tau), \tau) = \tilde{\Phi}(\boldsymbol{A}(\tilde{\boldsymbol{y}}(\tau_f)), \tau_f) + \int_{\tau_0}^{\tau_f} \tilde{L}(\boldsymbol{A}(\tilde{\boldsymbol{y}}(\tau)), \boldsymbol{B}(\tilde{\boldsymbol{y}}(\tau)), \tau)\mathrm{d}\tau \\ \text{s.t.}\ &\boldsymbol{\psi}_{0l} \le \tilde{\boldsymbol{\psi}}(\boldsymbol{A}(\tilde{\boldsymbol{y}}(\tau_0)), \boldsymbol{B}(\tilde{\boldsymbol{y}}(\tau_0)), \tau_0) \le \boldsymbol{\psi}_{0u} \\ &\boldsymbol{\psi}_{fl} \le \tilde{\boldsymbol{\psi}}(\boldsymbol{A}(\tilde{\boldsymbol{y}}(\tau_f)), \boldsymbol{B}(\tilde{\boldsymbol{y}}(\tau_f)), \tau_f) \le \boldsymbol{\psi}_{fu} \\ &\boldsymbol{g}_l \le \tilde{\boldsymbol{g}}(\boldsymbol{A}(\tilde{\boldsymbol{y}}(\tau)), \boldsymbol{B}(\tilde{\boldsymbol{y}}(\tau)), \tau) \le \boldsymbol{g}_u \\ &\boldsymbol{x}_l \le \boldsymbol{A}(\tilde{\boldsymbol{y}}(\tau)) \le \boldsymbol{x}_u, \quad \boldsymbol{u}_l \le \boldsymbol{B}(\tilde{\boldsymbol{y}}(\tau)) \le \boldsymbol{u}_u \quad \tau\in[\tau_0, \tau_f] \end{aligned} \tag{8}$$

where, $\tilde{\boldsymbol{y}}(\tau) = [\boldsymbol{y}(\tau), \dot{\boldsymbol{y}}(\tau), \ddot{\boldsymbol{y}}(\tau), \dots, \boldsymbol{y}^{(r+1)}(\tau)]^{\mathrm{T}}$. Differential flatness is an excellent property, especially for the hybrid PSO–LSM algorithm proposed in this paper. First, by the flat conversion the state equality constraints which are quite difficult to handle for PSO algorithm can be entirely eliminated as in Eq. (8). Thus, a feasible initial solution within the convergence domain of the NLP solver is probably obtained. Second, usually the number of the variables to be optimized can be dramatically decreased in this reformulated problem. Therefore, the following NLP