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Application of linear gauss pseudospectral method in model predictive control

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ABSTRACT

This paper presents a model predictive control(MPC) method aimed at solving the nonlinear optimal control problem with hard terminal constraints and quadratic performance index. The method combines the philosophies of the nonlinear approximation model predictive control, linear quadrature optimal control and Gauss Pseudospectral method. The current control is obtained by successively solving linear algebraic equations transferred from the original problem via linearization and the Gauss Pseudospectral method. It is not only of high computational efficiency since it does not need to solve nonlinear programming problem, but also of high accuracy though there are a few discrete points. Therefore, this method is suitable for on-board applications. A design of terminal impact with a specified direction is carried out to evaluate the performance of this method. Augmented PN guidance law in the three-dimensional coordinate system is applied to produce the initial guess. And various cases for target with straight-line movements are employed to demonstrate the applicability in different impact angles. Moreover, performance of the proposed method is also assessed by comparison with other guidance laws. Simulation results indicate that this method is not only of high computational efficiency and accuracy, but also applicable in the framework of guidance design.

1. Introduction

Model predictive control (MPC) is a form of control in which the current control is obtained by on-line solving a finite horizon open-loop optimal control problem, using the current state of the plant as the initial state in the calculation [1]. It has had a tremendous impact on industrial application development in the last decades. Now, an increasing number of researchers focus their attention on the development of “fast MPC”. Different from the off-line control policy, which is devoted to obtain the control by solving a full nonlinear optimal control problem, MPC usually involves employing the neighboring optimal control problem based on linearized dynamics. In general, a two-point boundary value problem (TPBVP) is formulated to calculate the current control. In order to reduce the consumption of time, Ohtsuka and Fujii have extended the stabilized continuation method to get a real-time optimization algorithm for nonlinear system [2]. A suitable continuation parameter is preselected to ensure satisfactory convergence. Lu Ping proposes a closed-form control law for trajectory tracking, in which a multi-step expansion is used to predict the state and Euler–Simpson approximation is employed to integral cost. Then, a quadratic programming problem is detected in analytically obtaining the current control [3]. Yan has applied the Legendre Pseudospectral method to solve the linear quadratic optimal control problem [4,5]. A set of linear algebraic equations transferred from the original problem can be easily solved to obtain the current control. And this method has been successfully applied in magnetic attitude

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Nomenclature

a_y, a_z	achieved lateral acceleration
a_{yc}, a_{zc}	commanded lateral acceleration
a_{yt}	target lateral acceleration
A, B	state-space matrices
dYN^2	convergence condition threshold
D	differential approximation matrix
D_m	aerodynamic drag acceleration
D^*	adjoint differential approximation matrix
f	dynamical equations
g	gravitational acceleration
H	Hamiltonian
J	cost function
m_m	mass of missile
n_{com}	longitudinal load profile commanded acceleration
N	number of LG node
Ne	proportional navigation guidance constant
P_f	terminal state positive semidefinite weighting matrix
Q	state positive semidefinite weighting matrix.
R	control positive semidefinite weighting matrix
S, K	constant matrices in linear algebraic equations
$Sref$	surface area of missile
t	flight time
t_τ	autopilot time constant
T_{trans}	preselected parameter to ensure fast transition in adaptive terminal guidance
u	control vector
v	Lagrange multiplier
Vm	missile velocity
Vc	closing velocity
x	state vector
$[x_m\ y_m\ z_m]$	missile position
$[x_t\ y_t\ z_t]$	target position.
X_k, U_k	trajectory information data
z	unknown vector in linear algebraic equations
γ_m	flight path angle of missile
$[\theta, \varphi]$	angles of the line-of-sight in XY plane and XZ plane.
λ	costate vector
λ_1, λ_2	adaptive proportional parameters
ρ	air density
σ	line of sight angle
$\dot{\sigma}$	line of sight rate
τ	normalized time
ψ	terminal constraint function
ψ_m	heading angle of missile
ψ_t	heading angle of target
Ψ	terminal constraint function
ω	gauss weights

Subscript

p	previous value
f	terminal value
m	missile
n	normalized value
t	target
$pitch$	pitch plane
yaw	yaw plane
0	initial value
*	normalizing variables

stabilization of satellite. It is noted that these methods are based on the assumption that the deviations from the reference trajectory are small. Paul Williams has proposed a new method to overcome this drawback [6]. This method approximates the original problem with successive linear approximation via quasi-linearization, and a Jacobi pseudospectral scheme is used to transfer the linear optimal control problem into solving a set of linear algebraic equations. An explicit integration for short horizon is used to overcome the comparatively large deviations. Another noteworthy point is that all these methods are receding horizon control that only solves a local optimal control from a finite short time. Therefore, the solution can be obtained quickly but is not globally optimal. Consequently, those methods are suitable for tracking a desired trajectory.

To solve the nonlinear optimal control problem with hard terminal constraints and quadratic performance index, a new nonlinear optimal control design called mode predictive static programming(MPSP) is proposed by Padhi [7–9]. This method combines the idea of nonlinear model predictive theory with the approximation dynamic programming. It can obtain the global optimal control. The current control is obtained via successively solving the static programming problem. However, this method should select a large number of nodes to ensure the Euler integration within a satisfactory tolerance.

In this paper, a method for solving the same problem of MPSP is presented. This method combines the philosophies of the nonlinear approximation model predictive control, linear quadrature optimal control and the Gauss Pseudospectral method [10]. Therefore, the proposed method is called Linear Gauss Pseudospectral Model Predictive Control(LGPMPC). Current control is obtained by successively solving a set of linear algebraic equations transferred from the original nonlinear control problem via linearization and the Gauss Pseudospectral method. This method is attractive from the point of view of computational efficiency, high accuracy with fewer discrete points and character that the solution can be expressed in a smooth function with control at discrete points. It is suitable for on-line implementation. The most notable difference from the Paul's method is that this method is used to solve the global optimal control problem with hard terminal constraints. Therefore, the horizon is not fixed; the linearization is not designed to track a desired trajectory. Another important difference is