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# On the stability and bifurcation analysis of dual-spin spacecraft

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ABSTRACT

The dynamics of dual-spin spacecraft under effects of energy dissipation are considered in this paper, where the damper masses in the platform ($\mathcal{P}$) and the rotor ($\mathcal{R}$) cause energy loss in the system. The Floquet theory is employed to obtain stability charts for different relative spin rates of the subsystem $\mathcal{R}$ with respect to the subsystem $\mathcal{P}$. Based on the general model for the system with nutation dampers on both $\mathcal{P}$ and $\mathcal{R}$, models are presented for a system whose nutation damper exists only in $\mathcal{P}$ as well as a system without nutation damper. The results obtained from the Floquet theory agree with the energy sink analysis in the literature. The bifurcation analysis based on the movement of loci of the Floquet multipliers as the system passes through the flutter stability boundary indicates that the system experiences the secondary Hopf (Neimark–Sacker) bifurcation. The investigations show that for spacecraft whose nutation damper exists only in one of the subsystems, there is no need to apply Floquet theory, and the Routh–Hurwitz criteria provides necessary and sufficient conditions for stability. Furthermore, for the case that only $\mathcal{P}$ has damping, the Lyapunov stability criteria agree with Routh–Hurwitz criteria.


## 1. Introduction

The subject of spinup dynamics and spin stabilization of spacecraft is of great significance in spacecraft design and space missions. A gyrostat model, consisting of a rigid body with an axisymmetric rotor, has often been used to study the dynamics of dual-spin spacecraft. Sandfry and Hall [1] considered a gyrostat model for spacecraft with single rotor and damper where the equilibria branches in the bifurcation diagrams were used for spinup dynamics analysis of the system. They [2] implemented the Lyapunov–Schmidt reduction and numerical continuation to design a criterion to avoid jump phenomena in an oblate gyrostat with damper.

The effect of energy dissipation on the motion stability of torque-free spinning bodies has been considered extensively in the literature. Assuming the mass of the dampers to be small with respect to the total spacecraft mass, Mingori [3] and Vigneron [4] implemented the Routh–Hurwitz (R–H) criteria to investigate the stability of dual-spin spacecraft with energy dissipation in both platform ($\mathcal{P}$) and rotor ($\mathcal{R}$). They further use the Floquet theory to study the stability of the system with damping in both subsystems. In the latter work [4], the averaging method is employed to solve the equations of motion of the system, and the results obtained are verified with the help of numerical solutions. The averaging method is used by Vigneron [5] to generalize the study for the case of multiple symmetric rigid bodies with damper devices, and the nonlinear differential equations of the system are obtained. Hall and Rand [6] used conservation of angular momentum and the method of averaging to reduce the spinup dynamics of dual-spin spacecraft to a simpler form. They further studied the accuracy of the reduced equations for different types of the spinup motion.

Following Mingori [3], the Lyapunov reducibility theory was applied by Guha [7] to reduce the linear system of

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differential equations with periodic coefficients to a kinematically similar system of autonomous differential equations. The energy sink analysis is employed by Likins [8] and Likins et al. [9] for the stability investigation of dual-spin spacecraft with energy dissipation and the possibility of obtaining asymptotic stable spin is proved for a body rotating about its minor axis. Different types of damping nonlinearities are considered in the latter [9]. Hughes [10] addressed the energy sink analysis to study spin stability and directional stability of gyrostats. Sen [11] considers energy dissipation on $\mathcal{R}$ as well as $\mathcal{P}$ in a model of symmetric dual-spin spacecraft, presents necessary and sufficient stability conditions, and shows that dual-spin spacecraft may have stable attitude even in the presence of some unstable characteristic roots in the system.

In this paper, we study the stability of dual-spin spacecraft with the influence of energy dissipation. The two subsystems $\mathcal{P}$ and $\mathcal{R}$ rotate about a common axis which can be the axis of either maximum or minimum inertia. The damper masses in both $\mathcal{P}$ and $\mathcal{R}$ can cause energy dissipation in the system. Using the concept of the energy sink analysis it follows that the minor axis can also be stabilized if $\mathcal{R}$ spins sufficiently fast. Using the R–H criteria, a preliminary approximation for the stability is provided based on Mingori [3]. Although this method cannot provide necessary and sufficient information about the stability behavior of the system, it can be used as a rough qualitative classification for the system with dissipation in both bodies. Then, Floquet theory is utilized for the linearized equations of motion and contour plots illustrating the linearized stability boundaries (as opposed to discrete sample points in the paper by Mingori [3]) are generated which describes the linearized stability of the system more precisely. Then, the full dimensionless nonlinear equations that govern the dynamics are used to obtain the bifurcation type for the system with nutation dampers on both $\mathcal{P}$ and $\mathcal{R}$, the movement of the Floquet multipliers is observed as the system becomes unstable, and the regions of attraction of the post-bifurcation behavior (Neimark–Sacker) are obtained numerically. The stability of spin about the axis of minimum inertia for the axisymmetric dual-spin spacecraft is studied for the initial conditions selected inside and outside the region of attraction. Next, simpler cases are considered where there are less than two nutation dampers in the system, and the results of the (exact) R–H criteria are compared with the literature. To the authors' knowledge, there has not thus far been a comparison of the results obtained by energy sink analysis and R–H criteria. Further, the full nonlinear equations of motion are utilized for a Lyapunov-based stability analysis. The stability conditions obtained using the Lyapunov criteria are shown to be consistent with those obtained using the R–H criteria.

## 2. Axisymmetric dual-spin spacecraft with nutation dampers on both $\mathcal{P}$ and $\mathcal{R}$

We first present the linearized equations of motion for an axisymmetric dual-spin spacecraft with nutation dampers on both $\mathcal{P}$ and $\mathcal{R}$. We refer the reader to the papers by Mingori [3] and Likins et al. [9] for a full derivation. Due to computational limits at that time, Mingori [3] studied this case by employing the Floquet theory on only a few points in the parameter plane, and therefore, the stability boundaries of the linearized system are not precisely shown.

A symmetrical configuration of spacecraft is illustrated in Fig. 1, where four mass-spring-dashpot dampers installed symmetrically on each cylinder are considered, only one of which is shown in the figure for convenience. $G$ is the mass center of the whole system. The dual-spin system has two rigid bodies $\mathcal{P}$ and $\mathcal{R}$ whose masses are $M$ and $M'$ disregarding the damper masses $m$ and $m'$ which are located at distances $a$ and $a'$, respectively, from axis $\hat{\mathbf{X}}_3$ (or $\hat{\mathbf{X}}'_3$). It is assumed that $\psi$ is the angle between the axes $\hat{\mathbf{X}}$ and $\hat{\mathbf{X}}'$. The angular velocity of $\mathcal{P}$ is given by $(\omega_1, \omega_2, \omega_3)$, and the positions of the damper masses are $z$ and $z'$. The stability of the system is considered about the reference motion

$$\omega_1 = \omega_2 = 0, \quad \omega_3 = \Omega, z = z' = 0, \quad \dot{z} = \dot{z}' = 0 \tag{1}$$

where $\Omega$ is the constant angular speed of $\mathcal{P}$ about $\hat{\mathbf{X}}_3$ (or $\hat{\mathbf{X}}'_3$) in the equilibrium state. The initial values of $\omega_1$, $\omega_2$, $\omega_3 - \Omega$, $z$, $\dot{z}$, $z'$, and $\dot{z}'$ are assumed to be sufficiently small for the validity of the linearized equations. It is then desired to determine the conditions under which the local asymptotic stability of the equilibrium solution in Eq. (1) is guaranteed. Construction of the equations of motion for the system with dampers on both bodies leads to time periodic linearized variational equations.

The general form of the full nonlinear dimensionless equations of motion is given in Appendix A.1, where the components of the state vector $\mathbf{x} \in \mathbb{R}^7$ are

$$x_1 = \frac{\omega_1}{\eta}, \quad x_2 = \frac{\omega_2}{\eta}, \quad x_3 = \frac{\omega_3}{\eta}, \quad x_4 = \frac{\dot{z}}{a\eta}, \quad x_5 = \frac{\dot{z}'}{a'\eta},$$

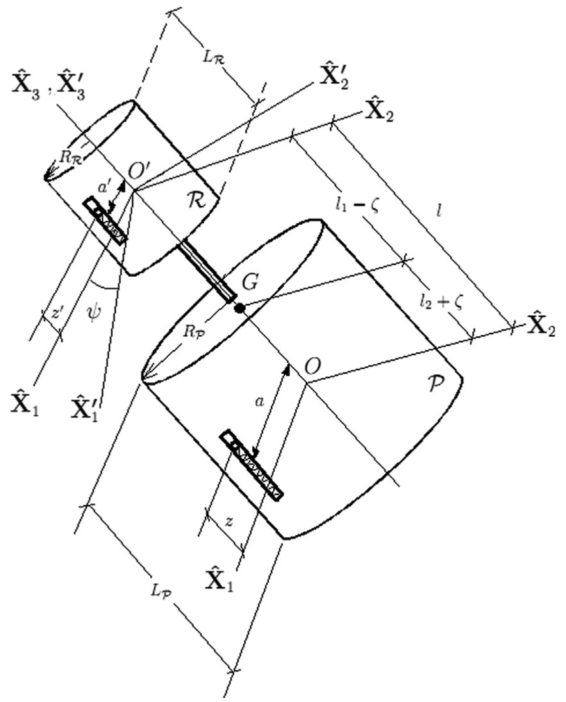

Fig. 1. Dual-spin model.