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High order optimal control of space trajectories with uncertain boundary conditions

P. Di Lizia [a,*], R. Armellin [a], F. Bernelli-Zazzera [a], M. Berz [b]

[a] *Politecnico di Milano, Dipartimento di Ingegneria Aerospaziale, Via La Masa 34, 20156 Milano, Italy*
[b] *Department of Physics and Astronomy, Michigan State University, East Lansing, MI, USA*

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ABSTRACT

A high order optimal control strategy is proposed in this work, based on the use of differential algebraic techniques. In the frame of orbital mechanics, differential algebra allows to represent, by high order Taylor polynomials, the dependency of the spacecraft state on initial conditions and environmental parameters. The resulting polynomials can be manipulated to obtain the high order expansion of the solution of two-point boundary value problems. Since the optimal control problem can be reduced to a two-point boundary value problem, differential algebra is used to compute the high order expansion of the solution of the optimal control problem about a reference trajectory. Whenever perturbations in the nominal conditions occur, new optimal control laws for perturbed initial and final states are obtained by the mere evaluation of polynomials. The performances of the method are assessed on lunar landing, rendezvous maneuvers, and a low-thrust Earth–Mars transfer.

1. Introduction

Nominal space trajectories are usually designed by solving optimal control problems that minimize the control action to meet mission constraints. However, uncertainties and disturbances affect the spacecraft dynamics in real scenarios. Moreover, state identification is influenced by navigation errors; consequently, the spacecraft state is only known with a given accuracy. Thus, after the nominal solution is computed, an optimal feedback control strategy that assures the satisfaction of mission constraints must be implemented. More specifically, given an initial deviation of the spacecraft state from its nominal value or a perturbation on the nominal final target conditions, the optimal control aims at canceling the effects of such errors by correcting the nominal control law, while minimizing propellant consumption.

Optimal feedback control was originally developed for linear systems. In linear optimal control theory, the system is assumed linear and the feedback controller is constrained to be linear with respect to its input [1]. The technological challenges imposed by the recent advances in aerospace engineering are demanding stringent accuracy requirements and cost reduction for the control of nonlinear systems. Unfortunately, the accuracy of linearized dynamics can drop off rapidly in nonlinear aerospace applications, affecting the performances of linear optimal controller. Thus, nonlinear optimal feedback control theory has gained interest in the past decades.

Various aspects of nonlinear optimal control have been addressed. Several techniques are available for solving control-affine problems, which are mainly based on dynamic programming or calculus of variations. In Bellmans dynamic programming, the problem is approached by reducing it to solving the nonlinear first-order partial differential Hamilton–Jacobi–Bellman (HJB) equation [2]. The solution to the HJB equation determines the optimal feedback control, but its use is very intricate in practical problems.

* Corresponding author. Tel.: +39 02 2399 8640.
E-mail address: pierluigi.dilizia@polimi.it (P. Di Lizia).

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An alternative approach is based on the calculus of variations and Pontryagins maximum principle, which show the Hamiltonian nature of the second order information of the optimal control problem [3]. Within this frame, the optimal control problem is reduced to a two-point boundary value problem (TPBVP) that is solved, in general, by successive approximation of the optimal control input using iterative numerical techniques. However, the solution determined is only valid for one set of boundary conditions, which prevents its immediate use for feedback control.

The complexity of finding the exact solution of the HJB equation has motivated research for approximated methods that are able to supply suboptimal laws for the control of nonlinear systems about reference solutions. In Bryson and Ho [2], an approximating technique is presented, based on a second order expansion of the augmented performance index of the optimal control problem, which is referred to as neighboring extremal paths computation. The State-dependent Riccati equation (SDRE) control method is among the more attractive tools to obtain such approximate solutions. It was originally proposed by Pearson [4], and Wernli and Cook [5], and then described in detail by Mracek and Cloutier [6], and Beeler [7]. This method involves manipulating the governing dynamic equations into a pseudo-linear non-unique form in which system matrices are given as a function of the current state and minimizing a quadratic-like performance index. An algebraic Riccati equation using the system matrices is then solved repetitively online to give the optimal control law. Thus, the SDRE approach might turn out to be computationally expensive when the solution of the Riccati equation is not properly managed. This can prevent its use for real-time optimal control. A significant computational advantage can be obtained with the $\theta{-}D$ technique [8]. Similar to SDRE, the $\theta{-}D$ technique relies on an approximate solution to the HJB equation. However, it offers a great computational advantage for onboard implementation without solving the Riccati equation repetitively at every instant.

Recent advances have been made in the frame of variational approach to optimal control theory. Second order methods were introduced by Bullock [9] and then extended by Olympio [10] to space trajectory design. Based on the Hamiltonian nature of the optimal control problem, the method computes a linear control update iteratively using the gradient of the Hamiltonian function. A higher order approach was introduced by Park and Scheeres [11] through the theory of canonical transformations. More specifically, canonical transformations solve boundary value problems between Hamiltonian coordinates and momenta for a single flow field. Thus, based on the reduction of the optimal control problem to an equivalent boundary value problem, they can be effectively used to solve the optimal control problem analytically as a function of the boundary conditions, which is instrumental to optimal feedback control. The main difficulty of this approach is finding the generating functions via the solution of the Hamilton–Jacobi equation. This problem was solved by Park and Scheeres by expanding the generating function in power series of its arguments.

Differential algebraic (DA) techniques [12] are used in this work to develop an alternative approach to the generating function method. Differential algebra serves the purpose of computing the derivatives of functions in a computer environment. More specifically, by substituting the classical implementation of real algebra with the implementation of a new algebra of Taylor polynomials, it expands any function f of v variables into its Taylor series up to an arbitrary order n. DA techniques are used in this work to represent the dependency of the spacecraft state on the initial conditions by means of high order Taylor polynomials. Then, the resulting Taylor polynomials are manipulated to impose the boundary and optimality conditions of the optimal control problem. This enables the expansion of the solution of the optimal control problem with respect to the initial conditions about an available reference trajectory. The resulting Taylor polynomials can be evaluated for new solutions of the optimal control problem, so avoiding repetitive runs of classical iterative procedures.

The paper is organized as follows. A brief introduction to differential algebra is given in Section 2. Being at the basis of the proposed methods, the possibility of expanding the flow of ODEs is presented in Section 3. The optimal control problem and the algorithm for the high order expansion of its solution are illustrated in Sections 4 and 5, respectively. The application of the algorithm to a rendezvous maneuver, a lunar landing, and a low-thrust Earth–Mars transfer problem is addressed in Section 6.

2. Differential algebra

DA techniques find their origin in the attempt to solve analytical problems by an algebraic approach [12]. Historically, the treatment of functions in numerics has been based on the treatment of numbers, and the classical numerical algorithms are based on the mere evaluation of functions at specific points. DA techniques are based on the observation that it is possible to extract more information on a function rather than its mere values. The basic idea is to bring the treatment of functions and the operations on them to the computer environment in a similar way as the treatment of real numbers. Referring to Fig. 1, consider two real numbers a and b. Their transformation into the floating point representation, $\overline{a}$ and $\overline{b}$, is performed to operate on them in a computer environment. Then, given any operation $\times$ in the set of real numbers, an adjoint operation $\otimes$ is defined in the set of FP numbers such that the diagram in figure commutes. (The diagram commutes approximately in practice, due to truncation

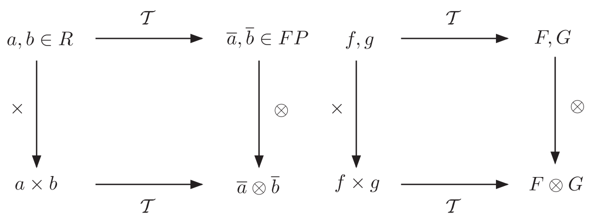

Fig. 1. Analogy between the floating point representation of real numbers in a computer environment (left figure) and the introduction of the algebra of Taylor polynomials in the differential algebraic framework (right figure).