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Nonlinear effects in the correlation of tracks and covariance propagation ☆

C. Sabol [a], K. Hill [b], K. Alfriend [c,*], T. Sukut [d]

[a] Air Force Maui and Optical Computing, Air Force Research Laboratory, USA
[b] Pacific Defense Solutions, USA
[c] Texas A&M University, USA
[d] USAF, USA

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ABSTRACT

Even though there are methods for the nonlinear propagation of the covariance the propagation of the covariance in current operational programs is based on the state transition matrix of the 1st variational equations, thus it is a linear propagation. If the measurement errors are zero mean Gaussian, the orbit errors, statistically represented by the covariance, are Gaussian. When the orbit errors become too large they are no longer Gaussian and not represented by the covariance. One use of the covariance is the association of uncorrelated tracks (UCTs). A UCT is an object tracked by a space surveillance system that does not correlate to another object in the space object data base. For an object to be entered into the data base three or more tracks must be correlated. Associating UCTs is a major challenge for a space surveillance system since every object entered into the space object catalog begins as a UCT. It has been proved that if the orbit errors are Gaussian, the error ellipsoid represented by the covariance is the optimum association volume. When the time between tracks becomes large, hours or even days, the orbit errors can become large and are no longer Gaussian, and this has a negative effect on the association of UCTs. This paper further investigates the nonlinear effects on the accuracy of the covariance for use in correlation. The use of the best coordinate system and the unscented Kalman Filter (UKF) for providing a more accurate covariance are investigated along with assessing how these approaches would result in the ability to correlate tracks that are further separated in time.

1. Introduction

One of the major challenges facing the future space surveillance system is the correlation of uncorrelated tracks (UCTs). A UCT is a track, a set of observations, that does not correlate to any object in the space object catalog. With the introduction of new sensors the catalog is expected to grow from its current size of about 20,000 objects to more than 100,000 objects. Before an object can be entered into the catalog, at least three UCTs must be correlated in order to develop a sufficiently accurate orbit. Thus, every new object entered into the catalog begins as three or more UCTs. Since the current UCT correlation process can be manually intensive a new automated process is needed for the development of the new catalog. One common technique for track association is to use fixed tolerances in position, or fixed gates. The fixed-gate track association technique described by Schumacher and Cooper [1] has two modes, "Verify" and "Identify." The verify mode is used when a track has already been tagged to a catalog object, and identification must be verified.

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* Corresponding author. Tel.: +1 831 648 1772.
E-mail addresses: keric.hill@pacificds.com (K. Hill), Alfriend@tamu.edu (K. Alfriend).

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First, the two orbit states to be compared are propagated to the same time, and rotated into the *RSW* coordinate system. The *R*-axis points along the position vector of the spacecraft from the planet to the spacecraft. The *S*-axis is in the orbit plane, normal to the *R*-axis, and the *W*-axis is normal to the orbit plane. To verify the identify of a tagged track, the limits of allowable position deviations in *RSW* coordinates are:

$$DR = \max(F, 0.003)$$
$$DS = 5DU$$
$$DW = 0.002a \tag{1}$$

where a is the semi-major axis. F is used for the along-track limit and is

$$F = 0.01 + 0.0006\log_{10}\left(\left|0.5\dot{n} + 10^{-21}\right|\right) \tag{2}$$

where $\dot{n}$ is the rate of change of the mean motion in radians per CTU, where 1 Canonical Time Unit= 13.44685108 min. The track tag is considered verified if the position difference in the radial direction is smaller than *DR*, and the along-track difference is smaller than *DS*, and the cross-track difference is smaller than *DW*. If the track does not match any pre-existing objects in the catalog, it is called an uncorrelated track, or UCT.

The identify mode is used for UCTs. An attempt is made to determine the most likely catalog object with which it should be identified. To identify a track with no tag, the track state is propagated to the epochs of all similar catalog states. The position difference is rotated into the *RSW* frame. If the radial, along-track, and cross-track differences are less than 20, 200, and 5 nautical miles respectively, then the quantity k_{gate} is computed:

$$k_{gate}^2 = \left(\delta\mathbf{X}^{RSW1}\right)^T M(\delta\mathbf{X})^{RSW1} \tag{3}$$

where $\delta\mathbf{X}^{RSW1}$ is the position difference in the object 1 *RSW* frame and M is given by

$$M = diag\begin{bmatrix} \frac{1}{A^2} & \frac{1}{B^2} & \frac{1}{C^2} \end{bmatrix} \tag{4}$$

and $A=4$, $B=40$ and $C=1$. The association with the lowest value of k_{gate} is accepted as the valid identification. Three or four UCTs must be associated before an object is entered into the catalog. Associating four UCTs is required in a high drag environment. In addition when this process does not work there is a manually intensive process involving plots of elements from which the analysts try to associate tracks.

The limitations with this current association approach include: (a) it does not consider the velocities, (b) it does not consider the uncertainty, and (c) the manual process requires a lot of analyst time. To build the new space object catalog of over 100,000 objects a new automated process is needed.

Alfriend [2] presented a new concept for track association using the statistical distance between the two tracks at a common time. This distance is called the Mahalanobis [3] distance and has also been suggested by Blackman [4]. The process is: Given the states $\boldsymbol{X}_1(t_1)$ and $\boldsymbol{X}_2(t_2)$ and covariances $P_1(t_1)$ and $P_2(t_2)$ of two tracks at their respective epochs, t_1 and t_2, propagate $\boldsymbol{X}_1$ and P_1 to t_2 and compute

$$k^2 = \delta\mathbf{X}^T P_{12}^{-1}\delta\mathbf{X}$$
$$P_{12} = P_1(t_2) + P_2(t_2)$$
$$\delta\mathbf{X} = \mathbf{X}_2(t_2) - \mathbf{X}_1(t_2). \tag{5}$$

Note that $\mathbf{X}$ is the state, not just the position, consequently this approach is considering the full state, the position and velocity. Eq. (5) assumes the covariances are uncorrelated. We call this covariance based track association (CBTA). The Mahalanobis distance k is the number of standard deviations. Reference [5] showed that the distribution function for k for a 6-dimensional state is

$$F(k) = 1 - \frac{1}{8}\left(k^4 + 4k^2 + 8\right)\exp\left(-\frac{k^2}{2}\right) \tag{6}$$

Fig. 1 shows $F(k)$ as a function of k and Table 1 provides specific values of $F(k)$ for various values of k. Thus, if $k=4$ then the probability of association is 98.6%.

Reference [2] showed that if the measurement errors are zero mean and Gaussian and the covariance represents the state errors, i.e., the linear propagation of the covariance is valid, then the error ellipsoid defined by Eq. (5) is the optimum association volume for the probability defined by k. Reference [2] also showed that the volume of the error ellipsoid is constant with time if there are no dissipation forces, e.g., atmospheric drag. Thus, the probability of association defined by k does not degrade as the time between tracks increases. However, even with no dissipation forces present as the time increases the neglected nonlinear effects in the propagation of the covariance will eventually start to have an effect and reduce the probability of association. The objectives of this project were to determine the best coordinate system in which to operate, to determine how long the linearity is valid in the various coordinate systems, and how to accommodate the nonlinearities. Section 2 discusses the

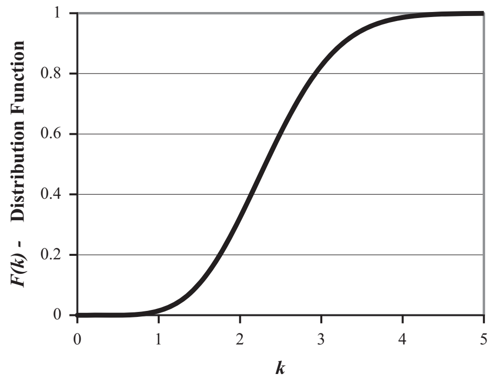

Fig. 1. $F(k)$ vs. k for the 6D state.

Table 1
$F(k)$ for various values of k.

k	1	3	4	6
$F(k)$	0.014	0.323	0.986	0.999997