Acta Astronautica 69 (2011) 429–439

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro

A mathematical model for the movement of food bolus of varying viscosities through the esophagus

Dharmendra Tripathi

Department of Mathematics, Birla Institute of Technology and Science Pilani, Hyderabad Campus, Hyderabad 500078, India

ARTICLE INFO

Article history:
Received 9 December 2010
Received in revised form
28 February 2011
Accepted 28 March 2011
Available online 21 April 2011

Keywords:
Mathematical model
Variable viscosity
Peristaltic wave
Esophagus
Mechanical efficiency
Trapping

ABSTRACT

This mathematical model is designed to study the influence of viscosity on swallowing of food bolus through the esophagus. Food bolus is considered as viscous fluid with variable viscosity. Geometry of esophagus is assumed as finite length channel and flow is induced by peristaltic wave along the length of channel walls. The expressions for axial velocity, transverse velocity, pressure gradient, volume flow rate and stream function are obtained under the assumptions of long wavelength and low Reynolds number. The impacts of viscosity parameter on pressure distribution, local wall shear stress, mechanical efficiency and trapping are numerically discussed with the help of computational results. On the basis of presented study, it is revealed that swallowing of low viscous fluids through esophagus requires less effort in comparison to fluids of higher viscosity. This result is similar to the experimental result obtained by Raut et al. [1], Dodds [2] and Ren et al. [3]. It is further concluded that the pumping efficiency increases while size of trapped bolus reduces when viscosity of fluid is high.

1. Introduction

Esophagus is a flexible muscular 18–26 cm long and 1.5–2.5 cm wide tube that spans from the mouth to the stomach. It is confined within two sphincters that work as an inlet and an outlet for food boluses and handle them with great control. The tube is collapsible and remains in the collapsed state until masticated food bolus knocks at the upper sphincter signaling the process to start. The nasal cavity is closed at that time. The esophagus gets activated and fully stretched; the sphincter slowly allows the bolus to pass into it. Once the bolus is inside the esophagus, the upper sphincter gets closed and the esophagus is almost totally occluded at the head of it. The wall at the tail of the bolus near the sphincter begins to contract and that at the head starts relaxing. This process continues till the bolus reaches the lower sphincter, knocks at it and gets delivered to the stomach with subsequent closure of the lower sphincter. Muscles in the wall of the esophagus work in a coordinated manner to push food and liquids down into the stomach. This muscular movement is called peristalsis.

Although peristalsis is well known mechanism for biologists, investigation of it from mechanical point of view began very late. Latham [4] ignited the investigation by analytical and experimental approaches in his work. Shapiro et al. [5] theoretically examined the peristaltic flow of viscous fluid with constant viscosity induced by sinusoidal wall propagation. They performed the analysis under assumption of long wavelength and discussed the phenomena of reflux and trapping during peristalsis. Brasseur et al. [6] studied the effect of peripheral layer for Newtonian fluids with different viscosities on peristaltic pumping. Misra and Pandey [7,8] reported the influence of peripheral layer on peristaltic flow of power-law fluid and casson fluid with different viscosities.

Viscosity is an important physical property of fluid, which plays a vital role during fluid flow. It describes a fluid's internal resistance to flow and may be thought of

---

E-mail address: dtripathi.rs.apm@itbhu.ac.in

0094-5765/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.actaastro.2011.03.017

as a measure of fluid friction. It deforms by either shear stress or tensile stress. It measures the thickness or internal friction of fluids. Thus, water is “thin”, having a lower viscosity, while honey, fruits juice, mixed vegetables soup, fruits jam and foods are “thick”, having a higher viscosity. Seeing the importance of viscosity, several workers [9–17] studied the effect of variable viscosities on peristaltic flow of Newtonian and non-Newtonian fluids in a uniform/non-uniform channels/tubes. They discussed the effect of magnetic field, endoscope and non-Newtonian parameters on the flow pattern with variable viscosities.

All of the investigations on peristaltic flow of Newtonian and non-Newtonian fluid with variable viscosities have been studied for infinite length tube but from physiological point of view it is less suitable. Li and Brasseur [18] have studied the peristaltic flow of viscous fluid with constant viscosity through finite length tube and they have discussed the importance of finite length tube and have compared their findings with that of Shapiro et al. [5]. They focused the study on both local and global dynamics. Its application is more suitable for physiological flow in human body because most of the physiological vessels are found to have finite length. The issue of local dynamics such as spatial-temporal variations in local stresses in terms of the motility and efficacy of the transport process was raised by Brasseur and Dodds [19]. They found close resemblance with the experimental findings of Dodds [2] and Ren et al. [3]. Misra and Pandey [20] who investigated similar phenomena for power-law fluids came to similar conclusions, although they did not discuss the experimental findings. Pandey and Tripathi [21–25] have recently worked for magneto-hydrodynamics, Maxwell, Jeffrey, Casson and micro-polar fluids. Another experimental observation for swallowing of food bolus with different viscosities has been observed by Raut et al. [1]. They found that amplitude of bolus pressure as well as that of the pharyngeal clearing contraction was increased within the hypo-pharynx with increasing viscosity of the food bolus. Keeping these facts into consideration, we intend to study the influence of variable viscosities on peristaltic flow of viscous fluids in finite length channel. This study mainly focuses on the swallowing of food bolus with different viscosities through the esophagus.

2. Mathematical model

The wave propagating along the channel walls is mathematically represented (cf. Fig. 1) as

$$\tilde{h}(\tilde{\xi},\tilde{t}) = a - \tilde{\phi}\cos^2\frac{\pi}{\lambda}(\tilde{\xi} - c\tilde{t}) \tag{1}$$

where $\tilde{h}$, $\tilde{\xi}$, $\tilde{t}$, a, $\tilde{\phi}$, λ and c represent the transverse vibration of the wall, axial coordinate, time, half width of the channel, amplitude of the wave, wavelength and wave-velocity, respectively.

The governing equations for incompressible viscous fluids with variable viscosities are given by

$$\rho\left(\frac{\partial}{\partial\tilde{t}} + \tilde{u}\frac{\partial}{\partial\tilde{\xi}} + \tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right)\tilde{u} = -\frac{\partial\tilde{p}}{\partial\tilde{\xi}} + 2\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\mu}(\tilde{\eta})\frac{\partial\tilde{u}}{\partial\tilde{\xi}}\right) + \frac{\partial}{\partial\tilde{\eta}}\left\{\tilde{\mu}(\tilde{\eta})\left(\frac{\partial\tilde{u}}{\partial\tilde{\eta}} + \frac{\partial\tilde{v}}{\partial\tilde{\xi}}\right)\right\} \tag{2}$$

$$\rho\left(\frac{\partial}{\partial\tilde{t}} + \tilde{u}\frac{\partial}{\partial\tilde{\xi}} + \tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right)\tilde{v} = -\frac{\partial\tilde{p}}{\partial\tilde{\eta}} + 2\frac{\partial}{\partial\tilde{\eta}}\left(\tilde{\mu}(\tilde{\eta})\frac{\partial\tilde{v}}{\partial\tilde{\eta}}\right) + \frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\mu}(\tilde{\eta})\left(\frac{\partial\tilde{u}}{\partial\tilde{\eta}} + \frac{\partial\tilde{v}}{\partial\tilde{\xi}}\right)\right\} \tag{3}$$

$$\frac{\partial\tilde{u}}{\partial\tilde{\xi}} + \frac{\partial\tilde{v}}{\partial\tilde{\eta}} = 0 \tag{4}$$

where $\tilde{u}$, $\tilde{v}$, $\tilde{\eta}$, ρ, $\tilde{p}$ and $\tilde{\mu}(\tilde{\eta})$ are the axial velocity, transverse velocity, transverse coordinate, fluid density, pressure and variable viscosity, respectively. The non-dimensional parameters are given by

$$\left.\begin{array}{lllllll} \xi = \frac{\tilde{\xi}}{\lambda}, & \eta = \frac{\tilde{\eta}}{a}, & t = \frac{c\tilde{t}}{\lambda}, & u = \frac{\tilde{u}}{c}, & v = \frac{\tilde{v}}{c\alpha}, & h = \frac{\tilde{h}}{a}, & l = \frac{\tilde{l}}{\lambda}, \\ \phi = \frac{\tilde{\phi}}{a}, & p = \frac{\tilde{p}a^2}{\mu c\lambda}, & Q = \frac{\tilde{Q}}{ac}, & \mu(\eta) = \frac{\overline{\mu}(\overline{\eta})}{\mu}, & \delta = \frac{a}{\lambda}, & \mathrm{Re} = \frac{\rho ac}{\mu} & \end{array}\right\} \tag{5}$$

where l, μ, Q, δ and Re are the length of channel, fluid viscosity, volume flow rate, wave number and Reynolds number, respectively. Introducing the non-dimensional parameters from Eq. (5) in Eqs. (1–4). Applying the

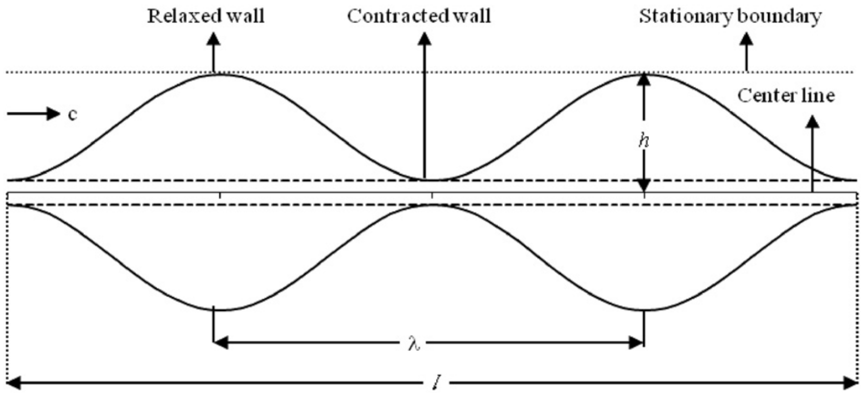

Fig. 1. Geometry of the peristaltic wave propagating along the channel.