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Input-to-state stability of model-based spacecraft formation control systems with communication constraints

Xi Liu*, K.D. Kumar

Department of Aerospace Engineering, Ryerson University, 350 Victoria Street, Toronto, ON, Canada M5B 2K3

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ABSTRACT

This paper investigates the formation keeping problem for multiple spacecraft in the framework of networked control systems (NCSs). A continuous-time representation of the NCS is considered for the tracking control of relative translational motion between two spacecraft in a leader–follower formation in the presence of communication constraints and system uncertainties. Model-based control schemes are presented, which employ state feedback (when the relative position and velocity vectors are directly measurable) and output feedback (when velocity measurements are not available), respectively, to guarantee input-to-state stability (ISS) of the system. The stability conditions on network transfer intervals are derived as simple eigenvalue tests of a well-structured test matrix. The results are then extended to include network communication delay. Numerical simulations are presented to demonstrate the effectiveness of the control scheme ensuring high formation keeping precision and robustness to nonlinearities and system uncertainties. The proposed controllers are robust not only to structured uncertainties such as system parameter perturbations but also to unstructured uncertainties such as external disturbances and measurement noises.

1. Introduction

Spacecraft formation flying (SFF), a novel concept of distributing the functionality of large spacecraft among several smaller and cooperative spacecraft, has been identified as an enabling technology for many space missions such as Earth observing, geodesy, deep space imaging and exploration, and in-orbit servicing [1,2]. It has received considerable attention due to several advantages including greater launch flexibility, higher system reliability, easier system upgrade and lower life cycle cost [3,4]. In particular, the tracking control problem of relative position is a topic of significant research interest in current reported literature. Several control approaches have been developed based on the linearized relative position dynamics known as Hill's equations [5] or Clohessy–Wiltshire equations [6], and nonlinear dynamics. Linear quadratic (LQ) control was adapted in [7–9] for formation keeping control. Queiroz [10] developed a nonlinear adaptive control law for the relative position tracking of multiple satellites. Wong et al. [11] designed an adaptive output feedback controller to guarantee the asymptotic convergence of relative translation errors. The tracking control problem has also been examined based on sliding mode approach [12–14] to provide asymptotically stable nonlinear tracking. The application of fuzzy technique was considered by Meng et al. [15], where a low-thrust fuzzy controller was presented on the basis of Clohessy–Wiltshire equations.

The aforementioned literatures assume perfect real-time communication. However, with the rapid developments in the microelectronics and telecommunication, communication network is adopted widely to exchange

* Corresponding author.
E-mail addresses: xiliu@ryerson.ca (X. Liu), krishnadevkumar@yahoo.com (K.D. Kumar).

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information (reference input, plant output, etc.) and control signals to make a formation exhibit a desired behavior. The insertion of a network into the control loop imposes communication constraints including irregularity of transfer intervals, existence of communication delay and the possibility of packet losses, among others. As a result of these specific constraints, direct application of many traditional control techniques may be impossible, or may lead to performance degradation and instability [16,17]. The traditional control results must be reevaluated before they can be applied to the networked spacecraft control systems. On the other hand, the spacecraft in a formation is always subject to uncertainties from system parameter perturbations, external disturbances, and so on, which may cause a serious drift of the relative positions. These factors all need to be considered for achieving desired formation control performance.

The objective of this paper is to develop robust and intelligent control algorithms for spacecraft formation keeping in the presence of communication constraints and system uncertainties. When a control loop is closed via a communication channel, the interconnection is referred to as a "networked control system" (NCS) [18–21,24–26]. Several specific control techniques have been developed to deal with communication constraints. One such technique is model-based control, which was used in [22,23] for sampled-data systems and in [24–26] for NCSs. In model-based NCSs, a plant model is included into the controller node and the control action is calculated based on the current state of the model rather than the actual state of the plant. Furthermore, the state of the model is updated from time to time based on measurement of the actual state of the plant. Essentially, the model-based approach utilizes a trade-off between two control strategies: closed-loop control which guarantees good performance characteristics but requires real-time data exchange, and open-loop control which requires little communication but lacks desired performance guarantees.

This paper focuses on input-to-state stability, proposed by Sontag [27], for the SFF system with disturbances. This notion is important given an intrinsic robustness problem. Indeed, the presence of disturbances (in addition to the model mismatch resulting from system parameter perturbations) implies that model estimates will deviate from the true plant output measured by sensors. In this paper the formation keeping problem for multiple spacecraft flying is investigated in the framework of networked control systems. A continuous-time representation of the NCS is considered for the tracking control of relative translational motion between two spacecraft in a leader–follower formation in the presence of communication constraints and system uncertainties. Model-based control schemes are presented, which employ state feedback (when the relative position and velocity vectors are directly measurable) and output feedback (when velocity measurements are not available), respectively, to guarantee input-to-state stability (ISS) of the system. The stability conditions on network transfer intervals are derived as simple eigenvalue tests of a well-structured test matrix. The maximum allowable transfer interval to guarantee formation stability can be determined as well. The results are then extended to include network communication delay. Numerical simulations are presented to demonstrate the effectiveness of the control scheme ensuring high formation keeping precision and robustness to nonlinearities and system uncertainties. The proposed controllers are robust not only to structured uncertainty such as system parameter perturbations but also to unstructured uncertainty such as external disturbances and measurement noises.

This paper is organized as follows. The dynamics of the leader and follower spacecraft are described in Section 2. In Sections 3 and 4, ISS stability conditions are developed for the cases of state and output feedback control, respectively. An extension to the state feedback system with communication delay is presented in Section 5. Numerical simulations are presented in Section 6. Finally, this paper is closed with conclusions in the last section.

2. Preliminaries

2.1. Notations and definitions

The following notation will be used throughout this paper. Denote $\mathbb{Z}^+=\{0,1,2,\ldots\}$. A continuous function $\alpha : R_{\geq 0}\rightarrow R_{\geq 0}$ is said to belong to class $\mathcal{K}$ if $\alpha(0)=0$ and it is strictly increasing. Also, a continuous function $\beta : R_{\geq 0}\times R_{\geq 0}\rightarrow R_{\geq 0}$ is said to belong to class $\mathcal{KL}$ if for each fixed $t\geq 0,\beta(\cdot,t)$ belongs to $\mathcal{K}$ and for each fixed $\sigma\geq 0,\beta(\sigma,t)$ decreases to zero as $t\rightarrow\infty$. The superscript T stands for the transpose of a vector or a matrix. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{m\times n}$ is the set of all real matrices of dimension $m\times n$. I stands for the identity matrix with appropriate dimension and diag$\{\cdots\}$ stands for a block-diagonal matrix. For a matrix A, we denote $\lambda_{min}(A)$ and $\lambda_{max}(A)$ as its minimal and maximal eigenvalues, respectively. The notation $|\cdot|$ refers to the Euclidean norm of a vector or the induced norm of a matrix. Given a continuous function $d:[t_0,\infty)\rightarrow\mathbb{R}^n$, we define its $\mathcal{L}_\infty$ norm as follows: $\|d\|_\infty=\sup_{\sigma\geq t_0}|d(\sigma)|$.

Definition 1 (*Sontag and Wang [27]*). The system $\dot{x}=f(x,d)$ is said to be input-to-state stable if there exist $\beta\in\mathcal{KL}$ and $\gamma\in\mathcal{K}$ such that, for all $d\in\mathcal{L}_\infty$ and $x(t_0)\in\mathbb{R}^n$ the solution of this system satisfies $|x(t)|\leq\beta(|x(t_0)|, t-t_0)+\gamma(\|d\|_\infty)$, $\forall t\geq t_0\geq 0$.

Remark 1. This inequality guarantees that for any bounded disturbances $d(t)$, the state $x(t)$ will be bounded. Furthermore, as t increases, the state $x(t)$ will be ultimately bounded by a class $\mathcal{K}$ function of $\|d\|_\infty$. Since, with $d(t)\equiv 0$, this inequality reduces to $|x(t)|\leq\beta(|x(t_0)|,t-t_0)$, ISS implies that the equilibrium point of the unperturbed system is uniformly asymptotically stable.

2.2. Relative dynamics

In this section we present the equations of motion of spacecraft in a leader–follower formation. The inertial coordinate system $S-XYZ$ is attached to the center of the Earth. Let r_l=$[r, 0, 0]^T$ denote the position vector of the leader spacecraft with respect to the inertial coordinate