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Laser radar based relative navigation using improved adaptive Huber filter

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ABSTRACT

An improved adaptive Huber filter algorithm is proposed to model error and measurement noise uncertainty in this work. The adaptive algorithm for model error is obtained by using an upper bound for the state prediction covariance matrix with augment of chi-square statistical hypothesis test in case of filter deteriorated by wrong residual information. The measurement noise is estimated at each filter step by minimizing a criterion function which was original from Huber filter. A recursive algorithm is provided for solving the criterion function. The proposed adaptive filter algorithm was successfully implemented in radar navigation system for spacecraft formation flying in high earth orbits with real orbit perturbations and non-Gaussian random measurement error. Simulation results indicated that the proposed adaptive filter performed better in robustness and accuracy compared with previous adaptive algorithms.

1. Introduction

For spacecraft formation flying in High Earth Orbit (HEO) with various orbit perturbations, a standard dynamic model can not accurately describe the real relative motion of the formation vehicles for navigation filter design. Some adaptive modifications should be augmented to the typical Kalman filter to incorporate process model uncertainty. By using theory of linear matrix inequalities [1], Subrahmanya and Shin [2] proposed an adaptive modification of Divided Difference Filter (DDF) which deals with model errors by estimating the upper bound of the state prediction in real time. Recently, it has been successfully adopted for adaptive relative navigation design for HEO formation scenarios [3].

Measurement problem arises for relative navigation implementation. Although Global Position System (GPS) provides a nature candidate for relative navigation and actually has been used as the major sensors in [3,4], the HEO formation spacecraft maintains visibility to at least one GPS satellite just 50% of the entire orbiting time (with assumed half angle of GPS antenna main lobe of typical 22.5 deg). HEO formation spacecraft will most likely involve the use of radar for relative navigation since the advantage of not relying the external sensors. It could be of great reliable for missions as autonomous rendezvous and docking in HEO or even away from HEO, perhaps in lunar orbit, for example. Because radar systems are known to exhibit non-Gaussian random measurement errors [5], it is important to develop estimation techniques for radar based relative navigation that are robust with respect to deviations from the assumption of Gaussianity. One such technique is the Huber filter [6,7], which is a combined minimum l_1 and l_2 norm estimator and it has actually been used for relative navigation design for robust rendezvous in elliptical orbit and tracking [8,9].

Extending previous research, this paper proposed an improved adaptive Huber filter by accounting for both model error and measurement noise uncertainty. The basic idea is to improve the adaptive algorithm similarly

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as in [2] by using the methodology given by Huber [6,7]. Moreover, a failure diagnosis method is augmented in this paper since it is also of great importance for practical applications. The method is basically a chi-square statistical hypothesis test for examining whether a random vector has an assumed mean and covariance [10,11]. Simulation results indicate that the proposed adaptive algorithm can be applied to relative navigation of spacecraft formation flying in HEO with promising robustness and accuracy.

2. Adaptive Kalman filter

Because of the complex nature of nonlinear estimation problems, many estimation algorithms rely on various assumptions to ensure mathematical tractability. We begin with an overview of the typical Kalman filter for states estimation. Then the Kalman filter algorithm is expanded and adapted for model uncertainty. The adaptive filter that incorporates unknown measurement noise is then provided.

2.1. Typical Kalman filter

The class of systems considered here is given as

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}) + G\mathbf{w}_{k-1}$$

$$\mathbf{y}_k = H\mathbf{x}_\mathbf{k} + \mathbf{v}_\mathbf{k} \tag{1}$$

where $\mathbf{x}_k \in \mathbb{R}^n$ denotes the state vector. The output vector is $\mathbf{y}_k \in \mathbb{R}^p$ and the nonlinear model is $f(\mathbf{x}_{k-1})$. We assume that f is either a globally Lipschitz continuous function or that it is locally Lipschitz continuous with $\mathbf{x}_k$ restricted to a compact domain $D \in \mathbb{R}^n$. The process/measurement noise $\mathbf{w}_k$ and $\mathbf{v}_k$ is assumed to be independent, identically distributed Gaussian random variables with

$$\mathbf{w}_k \sim \mathcal{N}(0,Q_k) \quad \mathbf{v}_k \sim \mathcal{N}(0,R_k). \tag{2}$$

The typical Kalman filter based estimation method is a recursive scheme that propagates a current estimate of a state and the error covariance matrix of that state forward in time. The filter optimally blends the new information introduced by the measurements with old information embodied in the prior state with a Kalman filter gain matrix. The gain matrix balances uncertainty in the measurements with the uncertainty in the dynamics model.

2.2. Adaptive Kalman filter for model error

The typical Kalman filter requires complete specifications of dynamical system to achieve optimal performance, i.e. the system model provided to the filter must be accurate. However, in practical situations, it is either unknown or partially known that seriously degrade the performance of the filter or even cause the filter to diverge. Some adaptive approaches should be done to compensates the effect of inaccurate dynamic model by rescaling the state covariance matrix during prediction. By using the theory of linear matrix inequalities introduced by Boyd et al. [1], Subrahmanya and Shin [2] have proposed an adaptive method to determine the upper bound for the DDF and demonstrated the superior performance as compared to the standard DDF. Applying the same method, this section provided the adaptive version of traditional Kalman filter.

Definition 1. For any two square matrices A and B, if $A-B$ is a positive definite (positive semi-definite) matrix, then we have $A>B$ ($A \geq B$). Moreover, A is an upper bound for B if and only if $A \geq B$.

Here we consider the modified state transition equation of (1) as

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}) + \Delta_f + G\mathbf{w}_{k-1} \tag{3}$$

where Δ_f is the bounded modeling error which assumed to satisfy $\|\Delta_f\|_\infty \leq \delta(\delta \geq 0)$.

Suppose we have the an upper bound of actual state covariance $P(0)$ at time instant zero, as $\hat{P}^u(0)$. Subrahmanya and Shin [2] have derived the upper bounds $\overline{P}^u(k)$ and $\hat{P}^u(k)$, i.e. the upper bound for *a priori* and *posteriori* state covariance at time instant k, sequentially given that $\hat{P}^u(k) \geq E[(\mathbf{x}_k - \hat{\mathbf{x}}_k)(\mathbf{x}_k - \hat{\mathbf{x}}_k)^T]$. The recursive adaptive procedure is given as:

Using states Eq. (3), the parametric form of the *a priori* state covariance upper bound can be given by

$$\overline{P}^u(k) = \lambda_k \overline{P}(k) + \alpha \,\mathrm{tr}(\hat{P}^u(k-1))I + \beta I \tag{4}$$

where $\overline{P}(k)$ is the state covariance prediction using the typical propagating equation. Coefficients λ_k, α and β are suitably defined parameters which can be calculated, and $\mathrm{tr}(\bullet)$ denotes the trace operation of matrix $(\bullet)$. Also I denotes identical matrix. The coefficients α and β may be considered offline tuning parameters while λ_k is adaptively estimated at each filter step by solving

$$\lambda_k = \min_{\varepsilon \geq 1} \varepsilon \tag{5}$$

such that $\varepsilon H\overline{P}(k)H^T + R_k + H(\alpha \,\mathrm{tr}(\hat{P}^u(k-1)) + \beta)H^T \geq \overline{P}_y^0(k)$ and $\overline{P}_y^0(k)$ is the unbiased estimation of the output covariance matrix, $\overline{P}_y(k) = H\overline{P}^u(k)H^T + R_k$, given by

$$\overline{P}_y^0(k) = \begin{cases} \gamma(1)\gamma(1)^T, & k=1 \\ \dfrac{\overline{P}_y^0(k-1) + \rho\gamma(k)\gamma(k)^T}{\rho+1}, & k>1 \end{cases} \tag{6}$$

The symbol $\gamma(k)$ denotes the innovation at time k. The parameter ρ determines the weighting given to current data in determining the noise covariance. Obviously, the unbiased estimation of $\overline{P}_y^0(k)$ in the form of Eq. (6) is susceptible to outliers in the innovations and a failure detection approach should be adopted in case of filter deteriorated by wrong residual information. Here we use an outlier rejection rule by means of chi-square statistical hypothesis test which is based on the confidence regions associated with filter residual, and compared it to a recomputed threshold.

Bar-Shalom [12] has demonstrated that when there is no failure of sensor measurement, then we have

$$\gamma(i) \sim \mathcal{N}(0, \overline{P}_y(i) = H\overline{P}^u(i)H^T + R(i)), \quad i = 2,3,4,\ldots,k \tag{7}$$

Moreover, a failure at time k will cause the expectation of $\gamma(k)$ to become non-zero and/or its covariance larger than $H\overline{P}^u(i)H^T + R(i)$. Thus, failure detection can be