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Laplace-resonant triple-cyclers for missions to Jupiter

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ABSTRACT

Cyclers are space trajectories that repeatedly encounter the same set of bodies indefinitely. Typically, cyclers are designed to encounter two bodies periodically, with only an occasional encounter with a third body. Because of the dynamics of the Laplace resonance in the Jupiter system, cycler trajectories that periodically return to three bodies are possible for Jupiter missions. Several cycler trajectories are proposed for purposes such as reducing mission length and increasing the number of flybys in a Jupiter system tour.

1. Introduction

Several authors have studied round-trip interplanetary missions that have led to the concept of cyclers [1–6]. One example of an interplanetary cycler mission that can travel between Earth and Mars indefinitely was designed by Byrnes et al. [7]. These cyclers, known as Aldrin cyclers, have the capacity to encounter either Earth or Mars up to 15 times in a 15-year period with a total ΔV of under 2 km/s. Later research has also been done to apply low-thrust capabilities to cycler design in order to reduce the flyby V_∞ at Earth and Mars [8,9].

Russell and Strange [10] extend the concept of cycler design from interplanetary cyclers to intermoon cycler trajectories. They focus on two-moon cyclers that would allow periodic return missions between any two of Jupiter's four Galilean moons and Titan–Enceladus cyclers in the Saturnian system. They propose a three-step model of designing these trajectories. They first find cycler trajectories in an ideal ballistic model assuming circular, coplanar orbits for the moons. They then optimize these ideal trajectories in a patched-conic ephemeris model to minimize ΔV. Finally, they integrate their patched-conic trajectories in a high-fidelity model.

We propose extending these two-moon cyclers to three-moon cyclers. We choose the Jupiter system for these proposed cyclers because all four of the Galilean moons have a high enough mass to permit gravity assist and the Galilean moons are all targets for scientific investigation. Additionally, a 1:2:4 orbital resonance exists among Io, Europa, and Ganymede called the Laplace resonance. We note that a Laplace resonance is defined as a 1:2:4 orbital resonance among three moons or planets [11–15]. The orbital resonance among Io, Europa, and Ganymede is the only Laplace resonance in the Solar System. All of the other orbital resonances in the Solar System only involve resonances between two moons (e.g. the 1:2 resonance between Dione and Enceladus). Hence, at least for near-term space missions, the mission design techniques which are introduced in this paper to exploit the Laplace resonance are uniquely applied to Jupiter system missions. (The only other known example of a Laplace resonance involves three exoplanets around the star GJ876, which is 15.4 light years away [16].)

The synchronicity of the Laplace resonance allows three-moon cyclers to have periods that are commensurate with

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the period of the Laplace resonance. Laplace-resonant cyclers reoccur indefinitely, so an entire Jupiter tour can be done while the spacecraft is in an Io–Europa–Ganymede cycler. In addition to indefinitely repeatable cyclers, we also propose cyclers that only reoccur once. These one-period three-moon cyclers contain two sets of three subsequent flybys and are useful for quickly reducing the orbital energy of a spacecraft in Jupiter orbit. These Laplace-resonant triple cyclers are found in an ideal model by using an extension of the Laplace resonance phase angles analysis proposed by Lynam et al. [11] The existence of these triple cyclers is confirmed by using the ephemeris models within AGI's STK (Satellite Tool Kit) [17] and JPL's MALTO (Mission Analysis Low Thrust Optimization) [18]. Since many of these triple-cycler trajectories have perijoves that are inside or near Jupiter's radiation field and ring plane, we also discuss the effects of these hazards on potential triple cycler spacecraft.

2. Laplace resonance phase angles analysis

2.1. Patched conic model

We perform the initial design of intermoon triple cyclers within the patched conic model [19–22]. This preliminary model does not incorporate the ephemerides of Jupiter's moons, so the moons are modeled to have circular, coplanar orbits that are constrained to be consistent with the Laplace resonance [12]. In our patched-conic model, the trajectory is modeled as a Keplerian two-body orbit around Jupiter with hyperbolic gravity-assists of the Galilean moons applied as instantaneous changes in the orbital elements of the two-body orbit about Jupiter. These gravity-assist flybys give mission designers the ability to modify the Jupiter-centered orbit of the spacecraft without expending ΔV. In the case of Laplace-resonant triple-cyclers, we design the three gravity-assists flybys of Io, Europa, and Ganymede to have a net effect of holding the semi-major axis and eccentricity of the Jupiter-centered orbit constant while shifting the argument of periapsis by 5.2°. We also need to ensure that the spacecraft can encounter all three moons in the same orbit. To this end, we must find the true anomaly of the spacecraft (both before and after each flyby) and the times of flight of the intermoon transfers. The true anomalies and times of flight are found by re-arranging the conic equation and Kepler's equation as follows [11]:

$$f_{moon} = -\cos^{-1}\left[\frac{-a_{moon}+a_{sc,in}(1-(e_{sc,in})^2)}{a_{moon}e_{sc,in}}\right] \tag{1}$$

where f_{moon} is the true anomaly of the spacecraft as it approaches the desired moon, a_{moon} is the semi-major axis of the desired moon's orbit, and $a_{sc,in}$ and $e_{sc,in}$ are the semi-major axis and eccentricity, respectively, of the spacecraft's orbit before the gravity-assist is modeled, and the time of flight between flybys is

$$T_{1,2} = \sqrt{\frac{a^3_{sc,out}}{\mu_J}}[(E_{moon,2}-e_{sc,out}\sin E_{moon,2}) -(E_{moon,1}-e_{sc,out}\sin E_{moon,1})] \tag{2}$$

where

$$E_{moon} = -\cos^{-1}[(a_{sc,out}-a_{moon})/(a_{sc,out}e_{sc,out})] \tag{3}$$

and where E_{moon} is the eccentric anomaly of a moon; $a_{sc,out}$ and $e_{sc,out}$ refer to the semi-major axis and eccentricity of the spacecraft during its transfer. The patched-conic model also requires that the orbital elements of an initial orbit are chosen and that the effects of all gravity-assists on the orbit are modeled.

2.2. The Laplace resonance

The Laplace resonance is a 1:2:4 orbital resonance that governs the motion Ganymede, Europa, and Io (i.e. Ganymede has double the orbital period of Europa and four times the orbital period of Io). The Laplace resonance constrains the relative positions of Ganymede, Europa, and Io with the following equation [11–14]:

$$180^\circ = 2\lambda_{Ga}-3\lambda_{Eu}+\lambda_{Io} \tag{4}$$

where λ_{Io} is Io's mean longitude, λ_{Eu} is Europa's mean longitude, and λ_{Ga} is Ganymede's mean longitude.

Lynam et al. [11] reformulated Eq. (4) to obtain the following phase angle relations among Ganymede, Europa, and Io:

$$\Delta\lambda_{Eu,Io} = 180^\circ+2\Delta\lambda_{Ga,Eu} \tag{5}$$

$$\Delta\lambda_{Eu,Io} = \{\pm 60^\circ, 180^\circ\}+2\Delta\lambda_{Ga,Io}/3 \tag{6}$$

$$\Delta\lambda_{Ga,Io} = \pm 90^\circ+3\Delta\lambda_{Eu,Io}/2 \tag{7}$$

$$\Delta\lambda_{Ga,Io} = 3\Delta\lambda_{Ga,Eu}+180^\circ \tag{8}$$

$$\Delta\lambda_{Ga,Eu} = \pm 90^\circ+\Delta\lambda_{Eu,Io}/2 \tag{9}$$

$$\Delta\lambda_{Ga,Eu} = \{\pm 60^\circ, 180^\circ\}+\Delta\lambda_{Ga,Io}/3 \tag{10}$$

where $\Delta\lambda_{Eu,Io}$ is the angle between the position of Europa and Io defined by the following relation:

$$\Delta\lambda_{Eu,Io} \equiv \lambda_{Io}-\lambda_{Eu} \tag{11}$$

We note that $\Delta\lambda_{Ga,Io}$ and $\Delta\lambda_{Ga,Eu}$ are defined analogously. The phase angle relations in Eqs. (5)–(10) allow the position (or two or three possible positions) of any one of the moons to be determined from the positions of the other two moons at any given time. Eqs. (5) and (8) have unique solutions, Eqs. (7) and (9) have two solutions, and Eqs. (6) and (10) have three solutions. The physical interpretation of the multiple solutions is that a moon could be in two or three possible positions if only the angle between the other two moons is known. Lynam et al. [11] apply this phase angle analysis to find Laplace-resonant triple-satellite-aided capture sequences that capture into orbit around Jupiter using gravity-assist flybys of Ganymede, Europa, and Io. In this paper, we use Eqs. (5)–(10) to find similar triple-gravity-assist sequences that repeat periodically once the spacecraft is already in orbit about Jupiter.

The dynamics of the Laplace resonance are useful for finding these triple-gravity-assist sequences that can form triple cyclers. The mean motion of each moon governs its dynamics by varying the mean longitudes over time. We write the mean longitudes of each moon as a function of